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MATHEMATICS

For Sixth form primary

Second Term

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غير مصرح بتداول هذا الكتاب خارج وزارة التربية والتعليم والتعليم الفنى

Revised by

Mr / Samir Mohamed Sedawy

Introduction

My dear pupils of sixth grade primary, it gives us pleasure to introduce this book to you as part of the developed mathmastics series. We were keen to provide many things for you on composing this book many things were taken in consediration in order to make studying mathematics an interesting popular and useful duty for you:

(1) Displying the topics in the easiest way and clearness using aproperiate language that adope with your information and experiences. So that it will help you to cope in the knowledge and ideas which were involved in each topic alon.

- The given ideas are listed gradually from the simplest to the hardest.
- We ensure forming the new concepts and ideas correctly before setting up associated operations via suitable activates.
- Linking the mathematical lessons with life through realistic Issues and problems in various applications hoping that you will fell the value of the mathematics and studying it as a useful in life.
- At many points within this book we give you opportunity to deduce ideas and reach information yourself depending on your experiences and thinking to grow up searching and self learning.
- At other points we invite you to work in groups with your colleagues to know their ideas and introduce to gather one part work.
- At other points too we call you to check the solution which were introduced to enrich your self confident and increase your ability for reach the correctness of things.
- The book was divided into units, the units were divided into lessons which involved with Images figures and illustrated diagrams. At the end of each lesson evaluated exercises were put, besides general exercises and unit test.

The book end contains model answers.

- The unit end contains activity to practice with your teacher help and you will find technological activity to deal with computer.

Finally... my dear pupil, in your classroom with your teacher and classmate, you should act posietily. Do not hesitate to ask questions. Trust that your participating will be appreciated, remember forever, mathematics involve many questions have more than one solution.

We ask allah that, we did well for our lovely Egypt.

Authors

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Unit One

Set of Integers

Lesson (1) : Set of Integers.

Lesson (2) : Comparing and ordering Integers.

Lesson (3) : Adding and subtracting Integers.

Lesson (4) : Multiplying and dividing Integers.

Lesson (5) : Repeated Multiplication.

Lesson (6) : Numerical Patterns.

- General Exercises on the unit one.
- Technological activity.
- Activity of the unit.
- Unit test.

1 Set of integers

- What do you learn from this lesson?
- Through your active participation, you will come to:
  - The concept of the set of integers.
  - The differentiation between the set of integers and the set of natural numbers.
  - The differentiation between the set of positive integers and the set of negative integers.
  - The relation between the subsets of the set "$\mathbb{Z}$".
  - The concept of the absolute value of the integer.

Mathematical Concepts

- The set of integers "$\mathbb{Z}$".
- The set of negative integers "$\mathbb{Z}^{+}$".
- The set of negative integers "$\mathbb{Z}^{-}$".
- The absolute value.

The need for more numbers

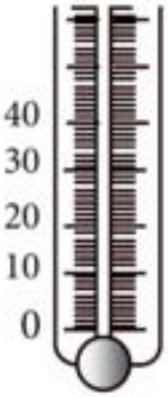
Think and discuss :

Opposite situations :

There are many opposite situations in our life. That can't be expressed through the set of natural numbers that you studied before such as:

1 - Temperature recorded in some cities such as: 6, 17, 25, 35 (possible in $\mathbb{N}$), while the two degrees C°, three or four .... under Zero,

(Is not possible in $\mathbb{N}$).



2 - Expressing the height of a residential building consisting of 12 floors above the ground (possible in $\mathbb{N}$), while the expression of three floors of the building underground.

(Is not possible in $\mathbb{N}$).

- Also according to the set of natural numbers that you have studied, the solution of the equation : $x + 5 = 7$ (possible in $\mathbb{N}$).

while $x + 7 = 5$ (Is not possible in $\mathbb{N}$).

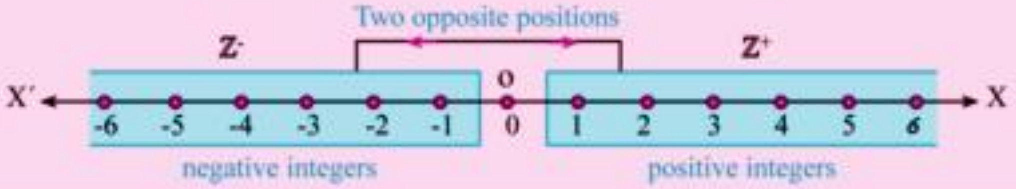
- To state that a city is at 150 metres above the sea level (possible in ℕ), while another city is at 200 metres below the sea level (Is not possible in ℕ).

from the previous, we deduce that :

Life is full of many examples of two opposite situations, one of them can be expressed in ℕ and the other can't be expressed in ℕ.

*** This means that the set of natural numbers is limited, in order to deal with the opposite phenomena of life, we has to expand (ℕ) in the other direction of the number line $\overrightarrow{ox'}$.**

*** It was agreed that the numbers after the Zero point (O) to the right ($\overrightarrow{ox}$) are positive (+), and the numbers on the left ($\overrightarrow{ox'}$) are negative (-), and are represented on the number line as the following:**



The numbers generated in this way are called (the set of integers), considering the numbers {+1, +2, +3, ...} are positive integers ($\mathbb{Z}^+$) and the numbers {-1, -2, -3, -4, ...} are negative integers ($\mathbb{Z}^-$).

This means that : $\mathbb{Z} = \mathbb{Z}^+ \cup \{0\} \cup \mathbb{Z}^-$

Example (1) : Write an integer to express each situation of the following:

1 - Hany gained LE 76 from his saving account.

2 - The temperature of Moscow City is 8 degrees below Zero.

3 - The depth of public garage consists of four floors underground in Cairo downtown.

4 - Paris rises 6 metres above sea level.

5 - Ahmed withdrew 6000 pounds from his bank account.

6 - The school added 10 marks to the student (Sarah), for her excellence in artistic activity.

- The Solution :

(1) (+76) (2) (-8) (3) (-4)

(4) (+6) (5) (-6000) (6) (+10)

** Representing the set of integers :*

1- The set of integers can be represented on the number line, without putting the sign (+) in front of the positive integers as it is understood and putting the sign (-) to express the negative integers.

Note that :

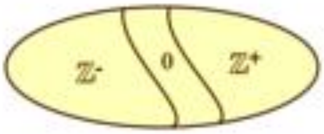
The set of integers is an infinite set and extends without limit from both sides.

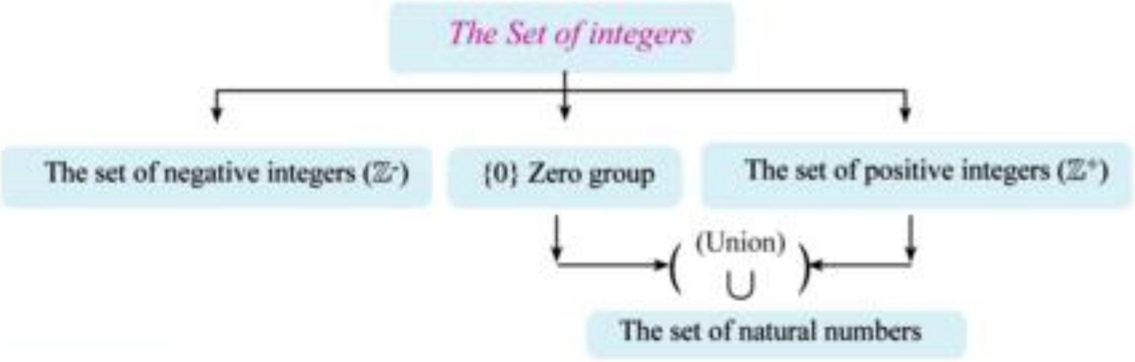
- Zero is neither positive nor negative.

- $\mathbb{N} \subset \mathbb{Z}$, $\mathbb{Z}^+ \subset \mathbb{Z}$, $\mathbb{Z}^- \subset \mathbb{Z}$, $\{0\} \subset \mathbb{Z}$.

2- $\mathbb{Z}$ Can be represented by the opposite Venn diagram.

3- $\mathbb{Z}$ Can be expressed through the following chart :




The Set of integers
The set of negative integers ($\mathbb{Z}^-$)
{0} Zero group
The set of positive integers ($\mathbb{Z}^+$)
(Union)
$\cup$
The set of natural numbers

Drill (1) :

Mark (true) or (false) and give the reason:

(a) Zero is smallest positive integer number. ( ) because : ....

(b) $\mathbb{Z} = \mathbb{Z}^+ \cup \mathbb{Z}^-$. ( ) because : ....

(c) $\mathbb{Z}^+$ is the set of counting numbers. ( ) because : ....

(d) $\mathbb{Z} = \mathbb{N} \cup \mathbb{Z}$: ( ) because : ....

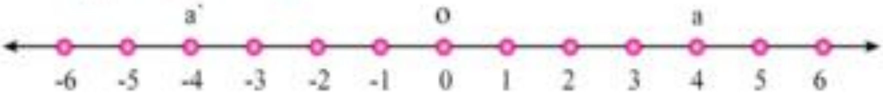
(e) $\mathbb{Z}^+ \cap \mathbb{Z}^- = \{0\}$. ( ) because : ....

The absolute value of the integer :

Think and discuss :

The absolute value of the integer (a) is the distance between the location of (a) and the location of Zero on the number line.
It is always positive and denoted by the symbol | a | .

- Not. throughr the following number line:



- The point (a) represents the number (4) and it is four units away from the point (o) representing "Zero".

- The point (a`) represents the number (-4) and it is four units away from the point (o) representing "Zero".

This means that | 4 | = 4, | -4 | = 4 .

We deduce that :

Each number and its inverse on the number line have the same absolute value because they are equidistant from the point of Zero (o).

Example (2) :

find the absolute value of : -3, 5, -12, -9, 0, 12.

the Solution : | -3 | = 3 , | 5 | = 5 , | -12 | = 12

| -9 | = 9 , | 0 | = 0 , | 12 | = 12

Drill (2) : Complete the following :

(a) | -102 | = … (b) - | -15 | = … (c) | -5 | + | 7 | = …

Solution :

(a) | -102 | = 102 (b) - | -15 | = -15 (c) | -5 | + | 7 | = 12

Drill (3) :

Express each of the following sets using the listing method, as in (a) :

(a) The set of integers which are less than 3. A = { 2, 1, 0, -1, -2, ...}.

(b) The set of integers which are greater than -2.

(c) The set of integers which are less than -5.

(d) The set of integers which are less than 6 and greater than -2.

(e) The set of integers between -4 and 3.

(f) The set of non - positive even integers.

Exercise (1 - 1)

(1) Complete the following using one of the words (positive - negative - Zero) :

(a) Moving forward is represented by .... numbers, while.
moving backward is represented by .... numbers.

(b) Moving from Zero point to the right is represented by .... numbers, while.
moving to the left is represented by .... numbers.

(c) - Lowering than sea level is represented by .... numbers,
- Height above sea level is represented by .... numbers.
- Sea level is represented by the number ....

(2) Represent the following numbers on the number line using one of the symbols (x) or (•) :

6 , -3 , 0 , -1 , 3 , 5 .

(3) Write the inverse of each of the numbers :

113 , -9 , 0 , 7.

(4) On the number line, colour each number of the following and its inverse with the same colour :

(a) 6 (b) -4 (c) -99

(5) Determine the value of the integer (b) in the following cases :

$|b| = 7$, $|b| = 16$, $|-9| = b$

(6) Find the value of x to get a true statement :

(a) $-5 \subset \{-1, 0, -3, x\}$
(b) $x \in \{2, 5, -3\} \cap \{-5, -2, -3\}$
(c) $\{2, x\} \cup \{-4, 0, 4\} = \{0, -2, 2, -4, 4\}$

(7) Mark (true) or (false) and give the reason :

(a) Zero $\in \mathbb{Z}^-$ ( ) because : ....
(b) $\varnothing = \mathbb{Z}^- \cap \mathbb{Z}$ ( ) because : ....
(c) $\mathbb{Z}^+ \cup \mathbb{N} = \mathbb{Z}^+$ ( ) because : ....
(d) $\{-17\} \in \mathbb{Z}$ ( ) because : ....

(8) Write each of the following sets using the listing method:

(a) **The** set of integers, that greater than (-2).

(b) **The** set of integers, that smaller than (-5).

(c) **The** set of integers between (-4), (3).

(d) **The** set of negative integers whose absolute value of each is greater than 4.

(9) Complete :

(a) $\mathbb{Z} = \mathbb{N} \cup$ ....

(b) $\mathbb{Z} \cup \mathbb{N} =$ ....

(c) $\mathbb{Z}^+ \cup \mathbb{Z} =$ ....

(d) $\mathbb{Z}^- \cup \mathbb{Z} =$ ....

(e) $\mathbb{Z}^+ \cap \mathbb{Z}^+ =$ ....

(f) $\mathbb{Z} =$ .... $\cup$ .... $\cup$ ....

(10) Put The suitable sign " $\in$, $\notin$, $\subset$ or $\not\subset$ " :

(a) -8 .... $\mathbb{Z}$

(b) 4.5 .... $\mathbb{Z}$

(c) $\frac{13}{5}$ .... $\mathbb{Z}$

(d) $\mathbb{N}$ .... $\mathbb{Z}$

(e) $\mathbb{Z}^+$ .... $\mathbb{N}$

(f) { 15 } .... $\mathbb{Z}^-$

(g) Zero .... $\mathbb{Z}^+$

(h) | -65 | .... $\mathbb{Z}^-$

2 *Ordering and Compring Integers*

- What do you learn from this lesson ?

- Through your active participation, you will come to :

- **The concept of ordering integers on the number line.**
- **Comparing between two integers.**
- **Ordering subsets of integers ascendingly and descendingly.**

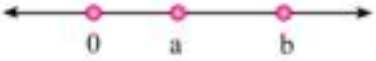
Mathematical Concepts

- **The ascending order in ℤ.**
- **The descending order in ℤ.**

Think and discuss :

Last year you studied the natural numbers and you have known that :

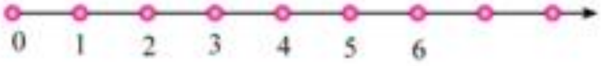
1- If the number (b) lies to the right of the number (a). then b is greater than a, and is written as ($b > a$).



If the number (a) lies to the left of the number (b), then a is less than b, and is written as ($a < b$).

The same property is available in the set of integers. (1).

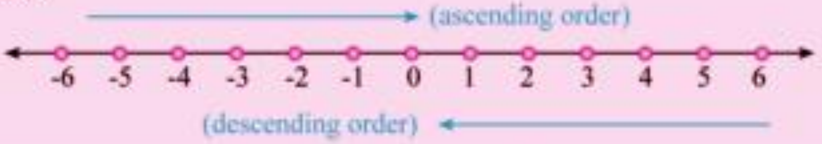
2- Property of sequence and the constant difference (unit) between any natural number and the next:



- The same property is also available in the set of integers. (2).

- from the above, we can deduce that :

(a) Both the set of natural numbers and the set of integers are arranged as shown on the following number line :



1- Arranged ascendingly (from the smallest to the greatest), If we move from left to right.

2- Arranged descendingly (from the greatest to the smallest), If we move from right to left.

(b) When you compare any two integers, the number which is located to the right of the other is the greater and vice versa.

This means that :

(1) ……. $-3 < -2 < -1 < 0 < 1 < 2 < 3 <$ ….. (ascending order).

(2) …….. $3 > 2 > 1 > 0 > -1 > -2 > -3$ ….. (descending order).

Example (1) :

Arrange the following numbers in an ascending order : -1 , 3 , 1 , -5 , 7.

The Solution :

The smallest number is - 5 (the first number at the left on the number line) followed by -1, 1 , 3, 7.

The ascending order is : -5 , -1 , 1 , 3 , 7.

Example (2) :

Put the correct sign (> , < or =) :

(a) -7 … -9
(b) 3 … -13
(c) -4 … 0
(d) $|-11|$ … 11
(e) -7 … $-|-5|$
(f) 30 … 103

The Solution :

(a) >
(b) >
(c) <
(d) =
(e) <
(f) <

Example (3) :

Write the previous integer and the next integer for each of the following integers :

(a) -7
(b) 15
(c) -23
(d) zero

Solution :

The integer number	The previous	The next
- 7	- 8	- 6
15	14	16
- 23	- 24	- 22
zero	-1	1

Exercise (1 - 2)

(1) Arrange the following integers :

(a) 6 , -60 , 2 , -17 , -22 , 0 (ascendingly).

(b) 1 , -11 , 3 , -1 , -8 , 5 (descendingly).

(2) Complete the space using suitable sign (>, < or =) :

(a) 3 … -6 (b) -7 … 17 (c) | -13 | … 3

(d) | -5 | … 5 (e) 3 + | -3 | … 8 (f) - | -4 | … 2

(3) Write the previous integer and the next integer for each of the following integers :

(a) -9 (b) 13 (c) 23

(4) Write the integers between each two integers of the following:

(a) -4 , 2 (b) -1 , 5 (c) -7 , 0

(5) Determine the Constant value by which the following integers increase, then complete the next three numbers :

(a) -7 , -6 , -5 , … , … , … (b) -2 , 0 , 2 , 4 , … , … , …

(c) -50 , -40 , -30 , … , … , …

(6) Write, suing the listing method each of the following sets :

- $X = \{x : x \in \mathbb{Z} \ , \ x < -3\}$
- $X = \{x : x \in \mathbb{Z} \ , \ x \leq -2\}$
- $X = \{x : x \in \mathbb{Z} \ , \ -1 \leq x < 5\}$
- $X = \{x : x \in \mathbb{Z} \ , \ -5 < x < \text{zero}\}$

3 Adding and subtracting Integers

- What do you learn from this lesson ?
- Through your active participation, you will come to:
* Possibility of addition in ℤ.
* Adding two positive or negative Integers.
* Adding two integers one of them is positive and the other is negative.
* Properties of addition operation in ℤ.
* Possibility of subtractin in ℤ.
* Subtracting two integers.
* Properties of subtraction operation on ℤ.

Mathematical Concepts
* Closure.
* Commutation.
* Additive - identity.
* Additive - inverse.
* Association.

First : Adding Integers :

Possibility of addition in ℤ :

Think and discuss :

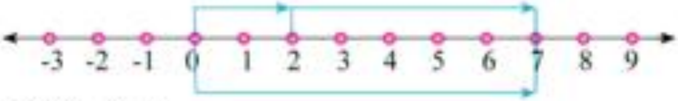
(a) Adding two positive integers :

Using the number line, we can add the two numbers 2, 5 as follows :

1- Start from Zero, and move right two unit to represent the number (2).

2- Start from (2) and move right five units to represent the number (5).

3- We arrive at the number (7) which is the sum.



Therefore : 2 + 5 = 7

i. e :

Adding positive integers and adding natural numbers are the same.

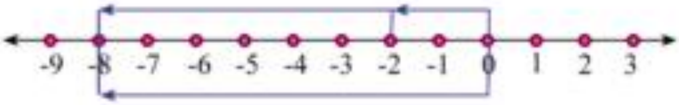
(b) Adding two negative integers :

Using the number line, we can add (-2) , (-6) as follows :

1- Start from Zero, and move left according to the absolute value of (-2).

2 - Start from (-2), and move left according to the absolute value of (-6).

3 - We arrive at the number (-8), which is the sum.



Therefore :

(-2) + (-6) = -8

i. e : **Adding two negative integers = negative integer**

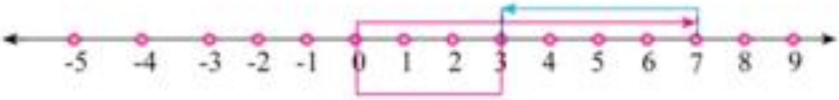
(c) Adding two integers on of them is positive and the other in negative :

Using the number line, we can add 7 + (-4) as follows :

1 - Start from Zero, and move right (7) unit to represent the number (7).

2 - Start from (7), and move left according to the absolute value of (-4).

3 - We arrive at the number (3) which is the sum.



Therefore :

7 + (-4) = 3

i. e : The sum of two different integers = positive or negative integer or Zero.

Example (1) :

Find the result of :

(a) (-6) + 6 (b) 4 + (-7) (c) 0 + (-4)

The Solution :

(a) (-6) + 6 = 0 (b) 4 + (-7) = -3 (c) 0 + (-4) = -4

Properties of addition operation in $\mathbb{Z}$:

- from the above: We deduce that the properties of addition operation on $\mathbb{Z}$ are :

1 - Closure property : $\mathbb{Z}$ is closed under the addition operation, this means that the sum of any two integers is an integer.

If $a \in \mathbb{Z}, b \in \mathbb{Z}$
Then $a + b = c \in \mathbb{Z}$

This means that : the addition operation is sometimes possible in $\mathbb{Z}$.

2- Commutative property :

The sum of any two integers doesn't change when commutating their positions.

If a, b are two integers, then : $a + b = b + a$

- for example : 6 + (-5) = (-5) + 6 = 1 , (-3) + (-2) = (-2) + (-3) = -5

you can verify this by using the number line.

3- The additive - identity :

Zero is the additive identity (neutral) in $\mathbb{Z}$ as It was in $\mathbb{N}$.

If a is an integer, then :

$$a + 0 = 0 + a = a$$

For example :

$7 + 0 = 0 + 7 = 7$, $(-8) + 0 = 0 + (-8) = -8$

4- The additive - inverse :

For each positive integer (a) on the number line, there is an opposite negative integer (-a), where Their sum = 0.

$$a + (-a) = (-a) + a = 0$$

Note that : the additive inverse of Zero is Zero

because $0 + 0 = 0$

The inverse of (-a) is $-(-a) = a$, this means : $-(-5) = 5$

For Example :

$4 + (-4) = -4 + 4 = 0$ (The inverse of 4 is (-4), and the inverse of (-4) is 4

5- Associative property :

The addition operation is associative in $\mathbb{Z}$ as it was in $\mathbb{N}$.

Note : To add three integers such as (-5 , 7 , 2),

We use associative property as follows :

$(-5 + 7) + 2 = \ldots + 2 = 4$

$(-5) + (7 + 2) = (-5) + \ldots = 4$

i. e. : $-5 + 7 + 2 = (-5 + 7) + 2 = (-5) + (7 + 2) = 4$

This means : If a, b, c are integers

Then : $a + b + c = (a + b) + c = a + (b + c)$

Note : Existing brackets means carrying out the operation inside the brackets first.

This property means that you can neglect the brackets and add any two numbers together.

Example (2) :

Use the properties of addition operation in $\mathbb{Z}$ to find the result of $(-17) + 19 + 17$.

State the property used in each step.

The Solution :

$(-17) + 19 + 17$	
$= (-17) + (19 + 17)$	(Associative)
$= (-17) + (17 + 19)$	(Commutative)
$= (-17 + 17) + 19$	(Associative)
$= 0 + 19$	(Additive inverse)
$= 19$	(additive identity)

Example (3) :

if $X = \{-2, 4, 2, -6\}$.

(a) What is the relation between X and the set of integers $\mathbb{Z}$?

(b) Show : Is X closed under the addition operation or not ?

The Solution :

(a) $X \subset \mathbb{Z}$ because each element in X belongs to (exists in) $\mathbb{Z}$.

(b) The idea : Add each two numbers together, if all results belong to X, then X is closed under the addition operation.

Therefore :

$(-2) + 4 = 2 \in X$, $(-2) + 2 = 0 \notin X$

$4 + (-6) = -2 \in X$, $2 + (-6) = -4 \notin X$

Therefore : X isn't closed under the addition operation.

Note : Only one result $\notin$ X is enough to make it non - closed.

Second Subtracting Integers :

Possibility of subtracting in $\mathbb{Z}$:

Think and discuss :

You have studied before the set of natural numbers, where you know that: $7 - 5 = 2$

Note : It can be written in a different form $7 + (-5) = 2$

and since $7 + (-5) = 2$, and from the relation between the addition and subtraction operations

we deduce that :

$2 - (-5) = 7$, this means $2 - (-5) = 2 + 5 = 7$

This means that :

subtracting two integers, a , b is defined as

a - b = a + the additive inverse of b . i.e. **a - b = a + (-b).**

Example (4) :

Find the result of each of the following :

(a) 9 - 5 (b) -7 – 4 (c) 6 - 11

The Solution :

(a) 9 – 5 = 9 + (-5) = 4 (b) -7 – 4 = -7 + (-4) = -11

(c) 6 - 11 = 6 + (-11) = -5

Example (5) :

(a) Find the result of : 5 - 8 , 8 – 5. what do you notice ?

(b) Find the result of : -9 - (3 – 8) , (-9 – 3) - 8. what do you notice ?

The Solution :

(a) 5 - 8 = 5 + (-8) = -3, 8 – 5 = 8 + (-5) = 3,

therefor 5 - 8 ≠ 8 – 5 (isn't commutative)

(b) (-9) - (3 - 8) = -9 - (-5) = -9 + 5 = -4

(-9 - 3) - 8 = -12 - 8 = -12 + (-8) = -20 ,

therefor -9 - (3 - 8) ≠ (-9 - 3) - 8. (isn't associative)

Properties of subtraction operation in ℤ :

From the above mentioned, we deduce that the properties of subtraction operation are :

1- Closure property : **ℤ is closed under the subtraction operation This means that the difference between any two integers is an integer.**

Therefore : the subtraction operation is always possible in ℤ.

2- Commutative property: **the subtraction operation in ℤ is not Commutative: a – b ≠ b – a for every a, b ∈ ℤ.**

From example (5) : **(a) where 5 - 8 ≠ 8 - 5**

3- Associative property: **The subtraction operation in ℤ is not associative : a – (b – c) ≠ (a – b) - c**

From example (5) : **(b) where -9 - (3 - 8) ≠ (-9 - 3) - 8 .**

Exercise (1 - 3)

1 - Use the number line to represent the following operations of addition and subtraction :

(a) $-3 - 3$ (b) $-5 + 7$ (c) $2 - (-3)$

2 - Write the integers representing in each of the following :

(a) $x < -1$ (b) $x > 7$ (c) $-4 < x < 4$

3 - Complete using the suitable sign $\in$, $\notin$, $\subset$, $\not\subset$

(a) $|-9| + 3 \ldots \mathbb{Z}$ (b) $\{9\} \ldots \mathbb{Z}$ (c) $\frac{3}{5} \ldots \mathbb{Z}$

(d) $\frac{9}{7+7} \ldots \mathbb{Z}$ (e) $\frac{6-6}{8} \ldots \mathbb{Z}$ (f) $\{-3 ; \frac{7}{11}\} \ldots \mathbb{Z}$

4 - Use the properties of addition operation in $\mathbb{Z}$ to find the result of the following :

(a) $-120 + 17 + 131$ (b) $2015 + 180 + (-1015)$

5 - Check the property of closure of the addition and subtraction on the following sets of numbers :

$X = \{-1 ; 0 ; 1\}$; $L = \{-2 ; -1 ; 0 ; 1 ; 2\}$

6 - Ramy deposited a sum of money amounting to LE 6220, then he withdrew an amount of LE 1211 and then deposited an another amount of LE 2110. How much is the balance of Ramy in the bank'

7 - A Submarine at a depth of 90 metres below sea level, rose 60 metres. Use the appropriate calculat to calculate the new depth of the submarine.

8 - Temperature is recorded in st. Catherine -3°C at three o'clock after midnight, while it is recorded 11°C. in the afternoon. Calculate the increase in temperature.

4 Multiplying and dividing integers

What do you learn from this lesson?

- Through your active participation, you will come to:

* Possibility of multiplying in $\mathbb{Z}$.
* Properties of multiplication operation in $\mathbb{Z}$.
* Possibility of dividing in $\mathbb{Z}$.
* Properties of division operation in $\mathbb{Z}$.
* Solving various exercises on multiplication and divison operations in $\mathbb{Z}$.

Mathematical Concepts :

* Multiplicative - identity
* Distributing multiplication over addition

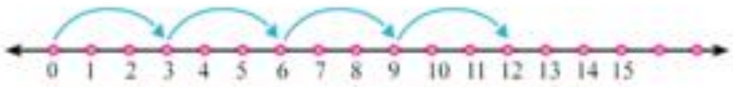
First : Multiplying Integers :

- Possibility of multiplying in $\mathbb{Z}$:

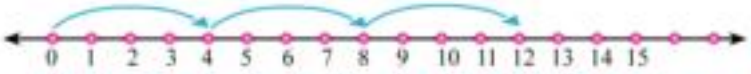
Think and discuss :

From your previous study, you knew that :

$3 \times 4 = 3 + 3 + 3 + 3 = 12 \in \mathbb{Z}^+$



$4 \times 3 = 4 + 4 + 4 = 12 \in \mathbb{Z}^+$

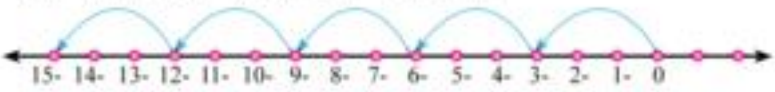


This means that :

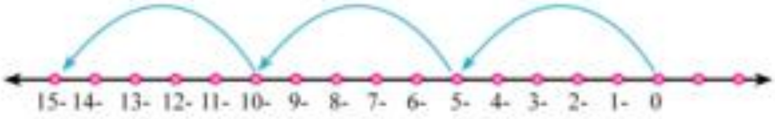
The product of two positive integers = positive integer.

* The same way :

(a) $(-3) \times 5 = (-3) + (-3) + (-3) + (-3) + (-3) = -15 \in \mathbb{Z}^-$



(b) $3 \times (-5) = (-5) + (-5) + (-5) = (-15) \in \mathbb{Z}^-$



This means that :

The product of two integers having different signs = negative integer.

(c) $(-2) \times (-4) = 8$, $(-3) \times (-5) = 15$

This means that :

The product of two negative integers = positive integer.

Example (1) :

Find the result of each of the following :

(a) $(-6) \times 3$ (b) $(-7) \times (-4)$ (c) $9 \times -(-8)$

The Solution :

(a) $(-6) \times 3 = -18$ (b) $(-7) \times (-4) = 28$ (c) $9 \times -(-8) = 9 \times 8 = 72$

Drill (1) :

Find the result of each of the following :

(a) $51 \times (-4)$ (b) $(-100) \times (-31)$ (c) $-(-5) \times -(-11)$

Properties of multiplication operation on $\mathbb{Z}$:

From the previous : We deduce that the properties of multiplication operation on $\mathbb{Z}$ are :

1 - Closure property : $\mathbb{Z}$ is closed under multiplication operation, this means that the product of any two integers is an integer.

i.e. The multiplication operation is always possible in $\mathbb{Z}$.

If $a \in \mathbb{Z}$, $b \in \mathbb{Z}$
Then $a \times b = C$, $C \in \mathbb{Z}$

2 - Commutative property : The multiplication operation is commutative in $\mathbb{Z}$.

If $a \in \mathbb{Z}$, $b \in \mathbb{Z}$ then : $a \times b = b \times a$

3 - The multiplicative identity : One is the multiplicative identity (neutral) in $\mathbb{Z}$ as It was in $\mathbb{N}$.

If $a \in \mathbb{Z}$, then : $a \times 1 = 1 \times a = a$.
For example : $9 \times 1 = 1 \times 9 = 9$, $(-7) \times 1 = 1 \times (-7) = -7$

4 - Associative property : the multiplication operation is associative in $\mathbb{Z}$ as it was in $\mathbb{N}$.

Note : To multiply three integers such as (-6, 8, -5), we use associative property as follows:

$(-6 \times 8) \times (-5) = (-48) \times (-5) = 240$

$(-6) \times (8 \times -5) = (-6) \times (-40) = 240$

i.e.: $(-6) \times 8 \times (-5) = (-6) \times (8 \times -5) = (-6 \times 8) \times (-5) = 240$

If a, b, c are three integers, then :

$$a \times b \times c = (a \times b) \times c = a \times (b \times c)$$

5 - The distribution :

It means distributing multiplication operation over addition operation.

Note : $5 \times (-3 + 7)$, $5 \times (-3) + 5 \times 7$

$= 5 \times 4$ $= (-15) + 35$

$= 20$ $= 20$

i.e. $5 \times (-3 + 7) = (5 \times -3) + (5 \times 7) = 20$

$$\text{If } a , b , c \in \mathbb{Z}, \text{ then: } a \times (b + c) = a \times b + a \times c$$

Example (2) :

Find the result of each of the following in two ways and state the used property :

(a) $6 \times (-2 + (-7))$ (b) $112 \times 17 + 112 \times -17$

- The Solution:

(a) $6 \times (-2 + (-7)) = 6 \times -2 + 6 \times -7$ (distributive property) $= -12 + (-42) = -54$

- Another solution :

$6 \times (-2 + (-7)) = 6 \times -9 = -54$ (adding the two numbers between the brackets, then multiply)

(b) $112 \times 17 + 112 \times (-17) = 112 \times 17 + (-17)$ (distributive property) (additive inverse)

$= 112 \times 0$

- Another solution :

$112 \times 17 + (-112 \times 17) = 0$ (Why?)

Second: Dividing Integers :

Possibility of dividing in $\mathbb{Z}$:

Note and discuss :

From your previous study, you knew that :

If $7 \times 5 = 35$, then : $35 \div 7 = 5$, $35 \div 5 = 7$

This means that :

Multiplication operation produces two division operations.

Similarly : If

$(-8) \times (-6) = 48$, then: $48 \div (-6) = -8$, $48 \div (-8) = -6$

$(-9) \times (+4) = -36$, then: $-36 \div 4 = -9$, $-36 \div (-9) = 4$

from the above, you can deduce that :

> **The quotient of two integers of the same signs is a positive number.**
>
> **The quotient of two integers having different signs is a negative number.**

Note :

All quotients in the previous cases belong to the set of integers, while the quotients of cases such as

$$\frac{8}{3},\ \frac{35}{9},\ -22 \div 5,\ \frac{-6}{-11} \notin \mathbb{Z}$$

Properties of division operation in $\mathbb{Z}$:

From the previous : We deduce that the properties of division operation in $\mathbb{Z}$ are :

1- Closure property : $\mathbb{Z}$ is not closed under division operation.

i.e. The division operation is not always possible in $\mathbb{Z}$.

2- Commutative property : The division operation is not commutative in $\mathbb{Z}$.

Note :

Division by zero is impossible in $\mathbb{Z}$ as it was in $\mathbb{N}$.

Example (3) :

Find the quotient of each of the following cases :

(a) $54 \div 6$ (b) $72 \div (-3)$ (c) $(-36) \div (-4)$

The Solution :

(a) $54 \div 6 = 9$ (b) $72 \div (-3) = -24$ (c) $(-36) \div (-4) = 9$

Drill (2) :

Find the quotient of the following two cases. What do you deduce ?

(a) $35 \div (5 \div 7)$ (b) $(35 \div 5) \div 7$

Drill (3) :

Find the value of x in each case of the following :

(a) $5 \times x = 45$ (b) $(-3) \times x = 27$

In the set of integers, Remember that:

- **The addition operation is always possible, closed, commutative and associative.**
- **The subtraction operation is always possible, closed, not commutative and not associative.**
- **The multiplication operation is always possible, closed, commutative and associative.**
- **The division operation is sometimes not possible, not closed, not commutative and not associative.**

Exercise (1 - 4)

1 - Find the result of each of the following :

(a) $(-131) \times (-3)$ (b) $5 \times (-4)$ (c) $(-8) \times 1$

(d) $(-9) \times 7$ (e) $0 \times (-11)$ (f) $-(-6) \times (-2)$

2 - Determine the possible division operation in $\mathbb{Z}$ of each of the following :

(a) $(-32) \div 8$ (b) $65 \div (-13)$

(c) $420 \div (-15)$ (d) $(-1300) \div 26$

3 - Find the result of each of the following in two ways :

(a) $(-4) \times [4 + (-1)]$ (b) $[5 + (-3)] \times (-11)$

(c) $6 \times (-6 + 0)$

4 - Find the value of x if :

(a) $8 \times x = -48$ (b) $x \times 9 = -45$

(c) $x \times (5 \times -13) = (-9 \times 5) \times (-13)$

5 Repeated multiplication

What do you learn from this lesson?

- Through you active participation, you will come to :

* The concept of repeated multiplication.
* the law of adding powers in multiplication.
* the law of subtracting powers in division.
* Solving various exercise on repeated multiplication.

Mathematical Concepts :

* Repeated multiplication.
* The base.
* The power.
* The n^{th} power of the number.
* The square of a number.
* The cube of a number.

Think and discuss :

Repeated multiplication means :

Multiplying the number by itself number of times.

For example:

$4 \times 4 \times 4$ means 4 is multiplied by itself three times.

- is written as (4^3) and is read as four to the power of three (4 is raised to the power 3).

- the number 4 is repeated and is called the base, while the number 3 is number of multiplying times and is called the power.

- (4^3) is called the third power of 4.

- Note :

$(4^3) = 64$, so 64 is called the third power of 4.

Similarly :

$(-3) \times (-3) \times (-3) \times (-3) = (-3)^4 = 81$ and $(-3)^4$ or 81 is called the fourth power of (-3).

Generally :

If a is an integer number, then :

$a \times a \times a \times a$ .... n-times $= a^n$, where $n \in \mathbb{Z}^+$

The basic laws of repeated multiplication :

First :The law of adding powers :

Note : $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^7$, It can be expressed as :

$$(2 \times 2) \times (2 \times 2 \times 2 \times 2 \times 2) = 2^2 \times 2^5 = 2^{2+5} = 2^7$$

$$(2 \times 2 \times 2) \times (2 \times 2 \times 2 \times 2) = 2^3 \times 2^4 = 2^{3+4} = 2^7$$

$$(2 \times 2 \times 2 \times 2) \times (2 \times 2 \times 2) = 2^4 \times 2^3 = 2^{4+3} = 2^7$$

$$(2 \times 2 \times 2 \times 2 \times 2 \times 2) \times 2 = 2^6 \times 2^1 = 2^{6+1} = 2^7$$

From the previous, we deduce that :

In case of multiplying the equal bases, we add the powers.

This means that :

If a $\in \mathbb{Z}$, a $\neq \mathbb{Z}$, a $\neq$ 0 then : $a^m \times a^n = a^{m+n}$ where m, n $\in \mathbb{Z}^+$

Second :

The law of subtracting powers :

Note : $3^5 \div 3^3 = \dfrac{3 \times 3 \times 3 \times 3 \times 3}{3 \times 3 \times 3} = 3 \times 3 = 3^2$

$$= \frac{3^5}{3^3} = 3^{5-3} = 3^2$$

From the above, we deduce that :

In case of dividing the equal bases, we subtract the powers.

This means that : If a $\in \mathbb{Z}$, a $\neq 0$

then : $\dfrac{a^m}{a^n} = a^{m-n}$ where m, n $\in \mathbb{Z}^+$, m > n

Example (1) :

Find the result of each of the following :

(a) $5^2 \times 5^3$

(b) $\dfrac{6^4 \times 6^5}{6^7}$

The Solution :

(a) $5^2 \text{ x } 5^3 = 5^{2+3} = 5^5 = 3125$

(b) $\dfrac{6^4 \times 6^5}{6^7} = \dfrac{6^{4+5}}{6^7} = \dfrac{6^9}{6^7} = 6^{9-7} = 6^2 = 36$

Note :

(1) The second power of any number is called the square of the number, for example :

8^2 is read as (8 to the power 2) or the square of 8.

(2) The third power of any number is called the cube of the number, for example :

7^3 is read as (7 to the power 3) or the cube of 7.

(3) The first power of any number = the number and isn't written, for example :

3^1 , 5^1 are 3 , 5.

(4) $(-3)^2 = (-3) \times (-3) = 9$

, $(-3)^3 = (-3) \times (-3) \times (-3) = -27$

We deduce that :

- If the base is a negative number raised to an even power, then the result is a positive number.
- If the base is a negative number raised to an odd power, then the result is a negative number.

Drill (1) : Complete the following table, as in the first case :

The number	The square of the number	The cube of the number	The fifth power of the number
2	$2^2 = 2 \times 2 = 4$	$2^3 = 2 \times 2 \times 2 = 8$	$2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32$
-1	....	....	$(-1)^5 =$ ....
3	....	....	....
-4	....	....	....

Example (2) :

Find the value of $\frac{3^4 \times (-3)^5}{3^7}$

The Solution:

$$\frac{3^4 \times (-3)^5}{3^7} = \frac{3^4 \times -(3)^5}{3^7}$$

$$= \frac{-(3^4 \times 3^5)}{3^7} = \frac{-3^9}{3^7}$$

$$= -(3)^{9-7} = -(3)^2 = -9$$

* The idea :
If the bases are different [3, (-3)] try to get equal bases

Note : $\frac{4^3 \times 4^4}{4^7} = \frac{4^7}{4^7} = 4^{7-7} = 4^0 = 1$

(the power of the numberator = the power of the denominator)

From the previous, we deduce that :

In dividion, if the powers are equal (m = n), then :

$$\frac{a^m}{a^n} = \frac{a^m}{a^m} = a^{m-m} = a^0 = 1, \text{ where } a \neq 0$$

this means that :

for every $a \in \mathbb{Z}$, $a \neq 0$, then : $a^0 = 1$

for example :

$5^0 = 1$, $(-17)^0 = 1$

Exercise (1 - 5)

1 - Find the value of each of the following :

(a) $(-7)^2$ (b) $(-5)^2 \times 2^2$

(c) $(-2)^4 + (-3)^3$ (d) $(-1)^{100} + (-1)^{101}$

(e) $(-4)^3 \times (-1)^5$ (f) $2^3 + 2^2$

2 - Find the result of each of the following :

(a) $3^7 \div 3^4$ (b) $(-6)^5 \div (-6)^3$ (c) $(-5)^5 \div 5^3$

3 - Arrange in an ascending order :

$(-2)^5$, $(-3)^2$, $(-4)^0$, $(-1)^{15}$, 3^2

4 - Find the result of each case of the following :

(a) $\dfrac{2^6 \times 2^5}{2^3 \times 2}$ (b) $\dfrac{(-3)^3 \times (-3)^4}{(-3)^5}$

(c) $\dfrac{(-8)^3 \times 8^4}{(-8)^7}$ (d) $\dfrac{9^6 \times (-9)^6}{(-9)^5 \times 9^2}$

5 - Arrange in a descending order :

10^2 , $(-1)^5$, 100^2 , $(-10)^3$, 1000000

6 - Put the suitable sign (> , < or =) :

(a) 4^2 8

(b) $(-6)^2$ .... - 12

(c) $(9)^2$ .... $(-3)^4$ (d) $\dfrac{1}{7^5} \times 7^5$ 1

6 Numerical patterns

- What do you learn from this lesson ?
- Through your active participation, you will come to :
  - Deducing the concept of numerical pattern.
  - Writing examples of numerical patterns in ℕ.
  - Describing pascal's triangle as one of the famous numerical patterns.
  - Deducing numerical patterns from pascal's triangle.
  - Describing the numerical pattern of various cases.

- Mathematical Concepts :
  - The numerical pattern.
  - Pascal's triangle.
  - The rule of the pattern.
  - Describing the pattern.

Note and think :

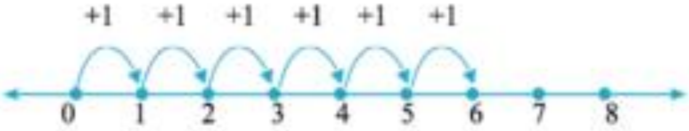
You have studied the set of natural numbers

ℕ = {0 ; 1 ; 2 ; 3 ; 4 ; 5 ; ....}

Perhaps you notice that the natural numbers (ℕ) represents a sequence of numbers according to a particular rule which is :

"Each number is more than its predecessor by one".

The following chart shows that .



For example :

The first number is 0, the second number is 1 consisting of 0 + 1 (through the arrow), the third number is 2 consisting of 1 + 1, the fourth number is 3 consisting of 2 + 1, the fifth number is 4 consisting of 3 + 1 .... and so on.

This sequence of numbers is called "numerical pattern"

- You have studied the subsets of the set of natural numbers (ℕ) such as :

The set of odd numbers = {1 , 3 , 5 , 7 , ...}

The set of even numbers = {0 , 2 , 4 , 6 , ...} and both are also a sequence of numbers according to the rule :

"Each number is more than its predecessor by 2".

And therefore any of them can be named "Numerical pattern".

- Numerical pattern : is a sequence of numbers according to a particular rule.

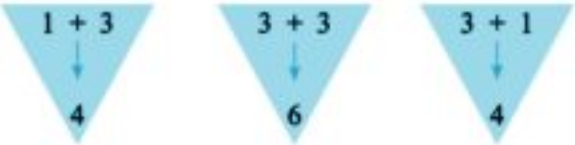
Pascal's triangle :

ascal's triangle is one of the well known numerical patterns.

Note : through the Pascal's triangle :

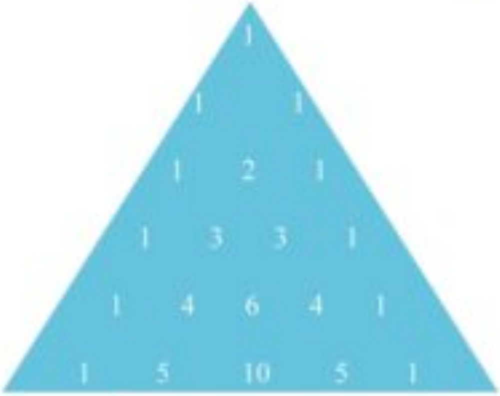
Each row begins and ends with number (one).

After the second row, we find that each number represents the sum of two numbers directly on top of it, its right and its left, for example : We find $1 + 3 = 4$, $3 + 3 = 6$, $3 + 1 = 4$ and are represented by the following triangles:



French physicist and mathematician lived in the seventeenth century in the period (1623 - 1662), He put the bases of the theory of probability in mathematics, invented the calculator which contributed to reaching modern computers, and in (1954) he presented a tripartite organization of numbers called Pascal's triangle as the following form :

Blaise Pascal



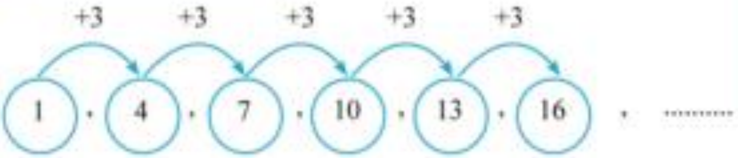
Blaise Pascal.

Drill (1) : Note the Pascal's triangle in the previous figure, and write the pattern of each of:

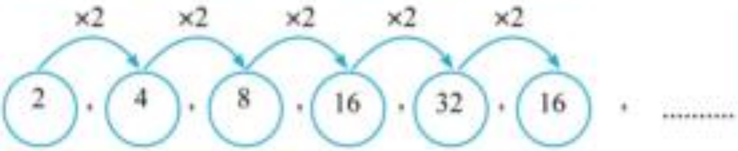
(a) The sum of numbers of the rows (b) the diagonals.

- **Describing of the pattern:** Means discovering the rule of the pattern and expressing it in words.

Note and discuss :

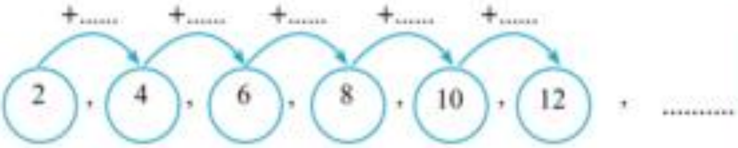


* description of the pattern : eac number is more than its predecessor by 3.



* description of the pattern: each number is twice of its predecessor.

Think and complete :



* description of the of the pattern : ....
....

Example :

Complete the following numerical patterns by writting three consecutive numbers:

(a) $\frac{1}{3}$, $\frac{1}{6}$, $\frac{1}{12}$, ... , ... , ...

(b) - 6 , -4 , -2 , ... , ... , ...

(c) 2 , 8 , 32 , 128 , ... , ...

Solution :

(a) $\frac{1}{24}$, $\frac{1}{48}$, $\frac{1}{96}$

(b) zero , 2 , 4

(c) 512 , 2048

Exercise (1 - 6)

1 - Complete the following table :

The numerical pattern	Description of the pattern
3 , 7 , 11 , 15 , 19 , 23 , ...	....
....	Each number is more than its predecessor by 5
$\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, 1 , $\frac{5}{4}$, ...	....
....	Each number is less than its predecessor by 4
3 , 9 , 27 , 81 , ...	....

2 - Complete the following numerical patterns by writing three consecutive numbers :

(a) 6 , 14 , 22 , 30 , 38 , ... , ... , ...

(b) $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$, ... , ... , ...

(c) 2 , 3 , 5 , 8 , 13 , ... , ... , ...

(d) 1 , 4 , 9 , 16 , 25 , ... , ... , ...

3 - Discover the rule of the numerical pattern and write the missing numbers in each case :

(a) 4 , 7 , ... , 13 , 16 , ... , ...

(b) 7 , ... , 15 , 19 , 23 , ... , ...

(c) 0.5 , 1 , ... , 2 , 2.5 , ... , ...

(d) 128 , 64 , ... , 16 , 8 , ... , ...

(e) .... , 15 , 12 , 9 , ... , ...

4 - An Egyptian land company reclaims 6 feddans per day to become prepared and ready for agriculture.

How many days do the company require to reclaim about 50 feedans ?

Write the numerical pattern which expresses this and describe it.

Drill (5) :

Write some other subsets of the set of natural numbers (ℕ) which represent "numerical pattern".

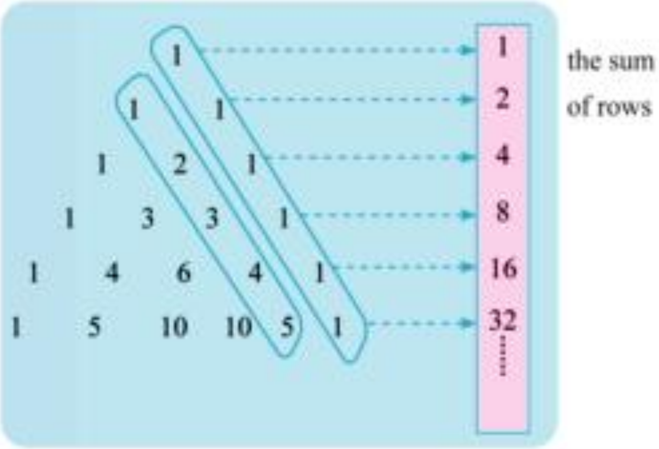
Drill (6) :

Copy the opposite Pascal's triangle in your notebook, and write the next two rows (in the same way).

Note :

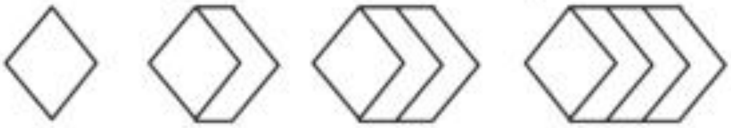
There are a lot of numerical patterns that we can get from Pascal's triangle, for example :

- The sums of numbers of the rows, as shown beside the triangle, represent a numerical pattern.
- The diagonals also represent numerical patterns.



Drill (7) :

Write the number of line segments below each shape, and then write the numerical pattern and describe it :



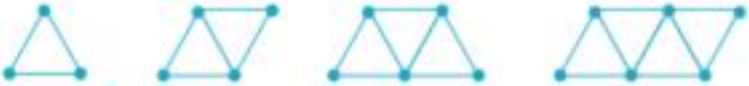
Number of line segments : ....

The numerical pattern : ....

Description of the pattern : ....

Drill (8) :

Write the number of triangles below each shape, and then write the numerical pattern and describe it :



Number of triangles : ....

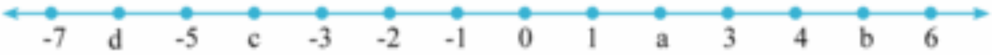
The numerical pattern : ....

Description of the pattern : ....

- Using the number of line segments, write another pattern and describe it.

General exercises on unit one

1 - Write the integers at the points a, b, c, d on the number line :



2 - Find the absolute value of the following integers :

-321 , 78 , -56 , -10 , 0 , 21

3 - Complete the following :

(a) $\mathbb{Z} \cap \mathbb{N}$ = ...
(b) $\mathbb{Z}^{+} \cap \mathbb{Z}^{-}$ = ...
(c) $\mathbb{Z} - \mathbb{N}$ = ...
(d) $\mathbb{Z} - \mathbb{Z}^{-}$ = ...
(e) $\mathbb{Z}^{+} \cup \{ 0 \}$ = ...
(f) $- | -45 |$ = ...
(g) the complement of $\mathbb{Z}^{-}$ with respect to $\mathbb{Z}^{-}$ = ...
(h) the complement of $\mathbb{Z}^{-}$ with respect to $\mathbb{Z}$ = ...
(i) the complement of $\mathbb{N}$ with respect to $\mathbb{Z}$ = ...

4 - Write the nearest integer to make the following statements true :

(a) $-4 >$ ....
(b) $2 <$ ....
(c) zero > ......
(d) $-6 <$ ....
(e) $| -6 | >$ ....
(f) zero < ......

5 - Complete in the same pattern :

(a) -20 , -18 , -16 , ... , ... , ...
(b) -15 , -10 , -5 , ... , ... , ...
(c) -4 , 0 , 4 , ... , ... , ...

6 - Arrange the following numbers in an ascending order :

(a) -9 , 17 , | -9 | , -15 , 16
(b) 3 , -30 , - | 8 | , 0 , 11

7 - Express each of the following sets using the listing method :

(a) the set of negative integers.
(b) the set of odd integers.
(c) the set of negative even integers.
(d) the set of integers between -3 and 13.

8 - Find the result of each of the following :

(a) $-12 + 7$
(b) $19 - (-11)$
(c) $-77 + (-3 + 77)$

9 - Find the result of each of the following :

(a) $-2 + 8$ (b) $-5 + 5$ (c) $-5 + (-2)$

10 - Complete to find the result of the following and state the property used in each step:

$116 + 190 + (-116) = 116 + (\ldots.. + -116)$ (……… Property)

$= 116 + (\ldots\ldots + 190)$ (……… Property)

$= (116 + \ldots..) + 190$ (……… Property)

$= \ldots\ldots\ldots + 190$ (……… Property)

$= 190$

11 - Check the property of closure of the addition and subtraction on the following set :

$X = \{-5 , 8 , 6 , -2\}$

12 - Find the result of each of the following in two ways :

(a) $(-6) \times [(-3) + 2]$ (b) $[7 + (-4)] \times 9$

13 - Find the value of m if : $-7 \times m = 42$

14 - Find the value of each of the following :

(a) $(-4)^2 \times 3^3$ (b) $(-1)^{30} + (-1)^{13}$ (c) $(-5)^3 \times (-1)^{17}$

(d) $2^{11} + 2^8$ (e) $(-4)^9 + (-4)^7$ (f) $(-3)^7 + 3^4$

15 - Complete the following table :

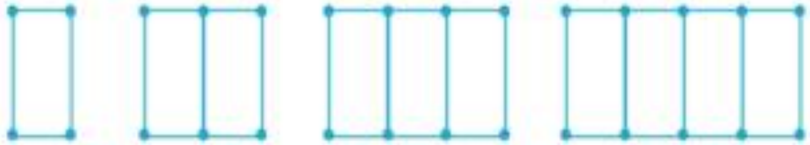
The numerical pattern	Description of the pattern
75 , 70 , 65 , 60 , 55 , …….	…………………………………
………………………	Each number is less than its predecessor by 4
1 , 10 , 100 , 1000 , ………	…………………………………
……………………………	Each number is twice its predecessor

16 - Find the result of each of the following :

(a) $\frac{(-5)^3 \times (-5)^2}{(-5)^4}$

(b) $\frac{(2)^5 \times (-2)^3}{(-2) \times 2^4}$

17 - Deduce the pattern rule expressing the following design, then write the numerical pattern :



Number of line segments : ....

The numerical pattern : ....

The pattern rule : ....

18 - Sheriff saves LE 51 every month. How many months does he need to save about LE 160?

Write the numerical pattern which expresses this and describe it.

19 - Complet :

(a) The Smallest positive integer is .... the greatest negative integer is ....

(b) The set of non-positive integers = ...

(c) The set of non-negative integers = ...

(d) .... is neither positive nor negative.

(h) The additive identity element in $\mathbb{Z}$ is .... , The multiplica tive identity element is ....

20 - Use commutative, Associative and distributive properties to find :

(a) $-74 + 65 + 74 + (-65)$

(b) $63 \times 85 + 63 \times 15$

(c) $54 \times 117 - 54 \times 17$

Technological activity :

Using the spreadsheet (Excel) program to find the sum and the product of two integers.

What do you learn of this activity ?

Using excel program in :

- Data entering (the set of integers) through Excel program.
- Calculating the sum and the product of two integers using the properties of Excel program.

Example :

Find the sum and the product of each two numbers of the following, then check the properties of each of addition and multiplication in $\mathbb{Z}$:

(a) 8 , 9 (b) -6 , 7 (c) -12 , 12

(d) -23 , -5 (e) 0 , 34

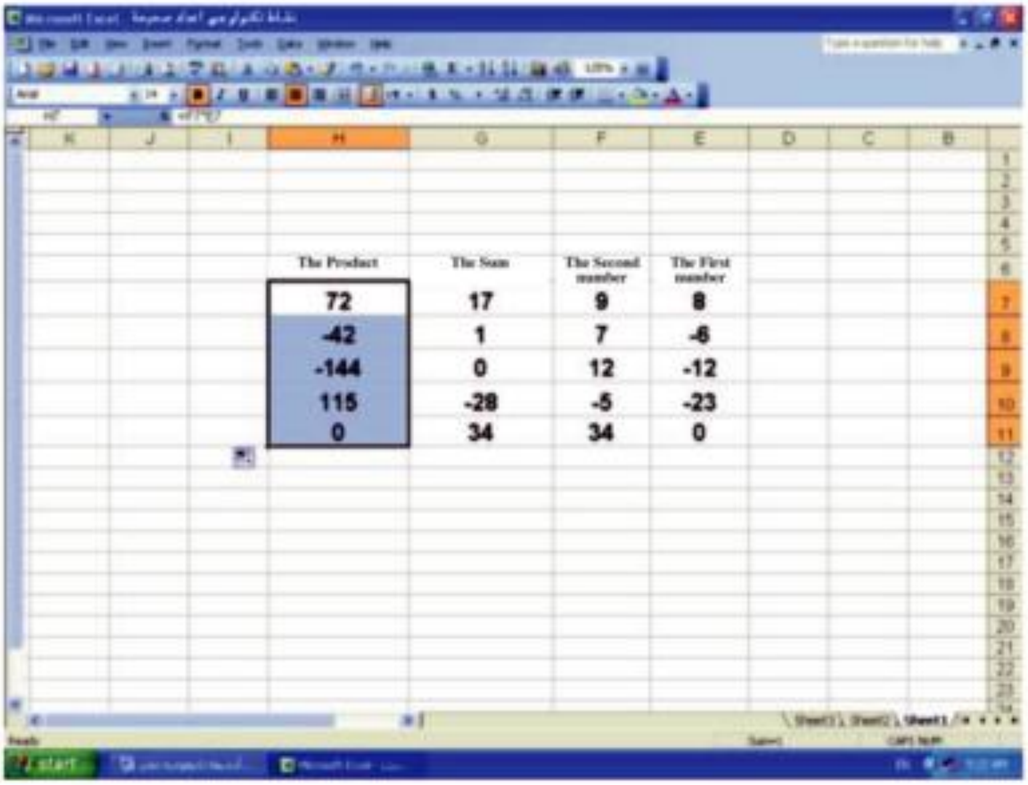
** The steps :*

(1) Click the (start) button ⟹ choose program ⟹ choose Microsoft Excel.

(2) Write the previous data in the selected cells on the program screen.

(3) To calculate the sum of the two numbers in row 7, select the cell G 7 and type (= F 7 + E 7), to calculate the product of them, select the cell H7 and type (= F7*E7), then chick the (Enter) button.

(4) To calculate the sum and the product of the numbers in the other rows, select the two cells H7, G7 then drag the "fill handle" to apply the properties of H7, G7 on the other cells, then the results will be shown as in the following figure :



The Product	The Sum	The Second number	The First number
72	17	9	8
-42	1	7	-6
-144	0	12	-12
115	-28	-5	-23
0	34	34	0

1 - Watch the weather forecast which describes the state of the weather in some cities, and register some cities of temperature less than zero and other cities of temperature greater than zero in the following table:

City						
Temperature						

- How many cities are of temperature less than zero ?
- Consider yourself a resident of one of the cities where temperature is greater than zero, and you will travel to the city of temperature less than zero.

(a) Calculate the difference in temperature between the two cities.

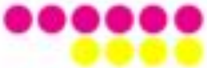
(b) Describe the preparations needed to travel to this city.

2 - Consider the counting balls as in the figure opposite, and then answer the following questions :

🔴 The red ball = +1
🟡 The yellow ball = -1
🟡🔴 The two balls = 0

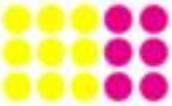
First : Write down the output of each process in each case of the following :

(a)



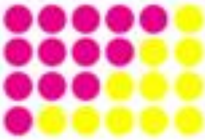
……………………………

(b)



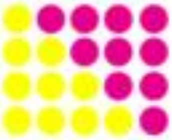
……………………………

(c)



……………………………

(d)



……………………………

Second : Express the following cases using the counting balls :

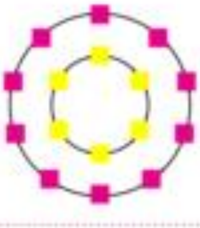
(a) -7 (b) 9 – 5 (c) -8 – 3 (d) 4 x -4

3 - Consider also the country balls as in the figure opposite :

The red square 🟥 = +1
The yellow square 🟨 = -1
The two squares 🟥🟨 = zero

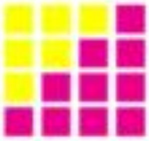
Express each figure using the suitable operation, then find the result :

(a)



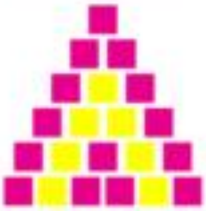
……………………………

(b)

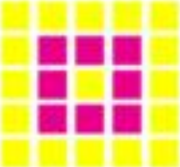


……………………………

(c)



(d)



Test of the unit

1 - Complete the following :

(a) The set of odd numbers $\cup$ the set of even numbers = ...

(b) $\mathbb{Z} = \mathbb{Z}^+ \cup$ .... $\cup$ ....

(c) .... Is the smallest positive number.

(d) $\mathbb{Z} = \mathbb{N} \cup$ ....

(e) $\mathbb{Z}^+ \cap \mathbb{Z}^- =$ ....

(f) $-|-54| =$ ....

(g) $\mathbb{Z}^-$ .... $\mathbb{N}$

(h) $\{15\}$ .... $\mathbb{Z}^-$

2 - Arrange the following numbers descendingly :

-9 , 0 , 7 , -15

3 - Represent the following addition and subtraction operations on the number line :

(a) $19 - |-9|$

(b) $4 - 6$

4 - Use the properties of addition and subtraction in Z to find the result of the following :

(a) $-5 + 8 - 15 =$ ....

(b) $-1 + 4 + 41 =$ ....

5 - One winter night, the TV announcer for the weather forecast referred to the temperature in Cairo 18°C and in Moscow -4°C. Calculate the difference in temperature between Cairo and Moscow. What would you advise the travellers from Cairo to Moscow.

6 - Find the result of the following :

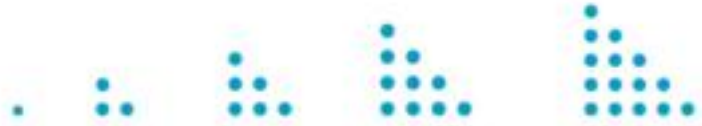
(a) $(-36) + (-4)$

(b) $2^3 \times (-1)^2 + 8$

(c) $\dfrac{(-4)^{11} \times 4^5}{4^{12}}$

7 - Khaled decided to lose weight at the rate of 3 kg monthly. If he is 90 kg heavy right now, then kow many months does he need to reach 69 kg? Write the expressing numerical pattern and describe it.

8 - Write the number of dots below each figure of the following :



The number of dots : ....

The numerical pattern : ....

The rule of the pattern : ....

Unite two

Equations and Inequalities

Lesson (1) : Equation and inequality of the First degree

Lesson (2) : Solving First degree equation in one unknown

Lesson (3) : Solving first degree inequality in on unknown

- *General exercises one the unit*
- *Technological activity*
- *Activity of the unit*
- *Unit test*

1 Equation and Inequality of the first degree

What do you learn from this lesson ?

Through your active participation you will come to :

- Concept of the equation.
- Concept of the inequality.
- Solving first degree equation in one unknown by substitution.
- Solving first degree inequality in one unknown by substitution.

Mathematical Concepts

- mathematical sentence.
- Closed sentence.
- Open sentence.
- The unknown.
- The degree of the equation.
- The inequality.
- Substitution set.
- Solution set.

(1) Concept of the equation :

Think and Discuss

Last year you have studied the mathematical sentences and you recognized its two types :

(a) Numerical sentences such as :

$3 + 9 = 12$, $13 - 7 = 6$, $3 \times 8 = 24$

(b) Symbolic sentences such as :

$9 - \square = 7$, $8 + x = 17$, $4 \times y = 24$

Notice that :

The numerical sentences are called closed mathematical sentences because we can determine whether they are right or wrong, while the symbolic sentences are called open mathematical sentences because we cannot determine whether they are right or wrong due to the existence of symbols like (□ or x or y) which are unknowns. When we replace the symbol by its numerical value the open sentence becomes a closed sentence. For example, in the symbol sentence :

$8 + x = 17$, if we replace x by 9, we get

$8 + 9 = 17$ (Closed sentence)

The mathematical sentences which contains the symbol " = " is called equations.

The equation : **is a mathematical sentence includes equality relation between tow sides.**

From the definition, we get :

1 - The equation has two sides including the relation (=) between them. F.e. $x + 1 = 7$ is an equation, its left hand side is ($x + 1$) and its right hand side is (7).

2 - In the equation :

$x + 1 = 7$, the symbol (x) is called (the unknown) which we want to know its value.

Example (1) :

Determine whether each of the following represents an equation or not tell why :

(a) $x + 5$ (b) $9 - 5 = 4$ (c) $x + 7 = 12$

The Solution :

(a) $x + 5$ is not an equation, because it does not contain equality of two sides.

(b) $9 - 5 = 4$ is an equation, because it contains equality of two sides.

(c) $x + 7 = 12$ is an equation, because it contains equality of two sides.

The concept of Inequality

Think and Discuss

1 - In figure (1) : There is a balance in the equality position. In its right pan there is a sack, has an unknown number of applex (x) + 2 apples and in its left pan there are 6 apples, we express this position by the equation : $x + 2 = 6$.

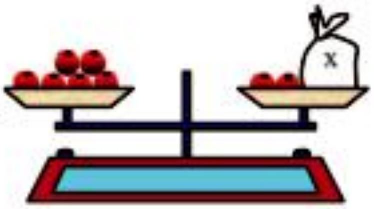


Figure (1)

2 - In figure (2) : 3 apples are added to the right pan to become $(x + 5)$ greater than the left pan (6 apples) we express this case by the mathematical sentence : $x + 5 > 6$.

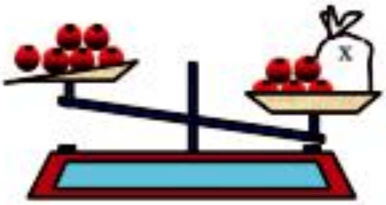


Figure (2)

3 - In figure (3) : 4 apples are added to the left pan as in the figure and we express this case by each of the two mathematical sentences :

$x + 5 > 6$, $x + 5 < 10$. Each of these sentence, is called inequality due to the existence of the inequaluity sign between the two sides.

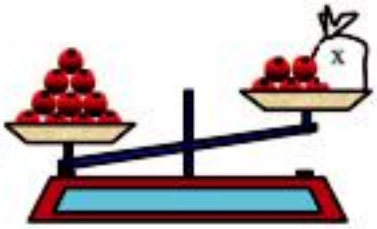


Figure (3)

The inequality is a mathematical sentence including the sign of inequality between two sides.

Example (2) :

Determine which of following represents an equation or an inequality giving reason :

(a) $x - 5 > 3$ (b) $x - 17$ (c) $2x < 7$

Solution :

(a) $x - 5 > 3$ is an inequality, because it contains the inequality sign between its two sides.

(b) $x - 17$ is neither equation nor inequality, because it does not contain the signs of equality or inequality.

(c) $2x < 7$ is an inequality, because of the existence of the inequality sign between its two sides.

Degree of the equation

The degree of the equation is equal to the highest power of the unknown (symbol) in this equation.

For example :

$x + 5 = 7$ is an equation of the first degree in one unknown (x).
, $x^2 + 3 = 8$ is a second degree equation in one unknown (x).
, $4x^3 - x = 29$ is a third degree equation in one unknown (x).

This year we will study only the equations and inequalities of the first degree in on unknown.

Solution of the equation or the inequality :

By solving the equation or the inequality, we mean getting the value (s) of the unknown in the equation or the inequality, so we need what is called the substitution set which is a set of integers, its elements can be used to replace the unknown (symbol) in both sides of the equation or the inequality to test whether its true or not.

The set of elements of the substitution set which verifies the equality of the two sides of the equation is called the solution set.

Example (3) :

Given that the substitution set is $L = \{0, 1, 2, 3\}$, find the solution set of :

(first) the equation $x + 3 = 5$ (second) the inequality $x + 3 < 5$.

Solution :

(first) the equation $x + 3 = 5$:

Substitute by the elements of L in the left hand side $(x + 3)$ to determine the elements which verify the equation as follows :

At $x = 0$, $0 + 3 = 3 \neq 5$, so "zero" does not verify the equation.

At $x = 1$, $1 + 3 = 4 \neq 5$, so "1" does not verify the equation.

At $x = 2$, $2 + 3 = 5 = 5$, so "2" verifies the equation.

At $x = 3$, $3 + 3 = 6 \neq 5$, so "3" does not verify the equation.

We get the solution set = $\{2\}$. Notice that $\{2\} \subset \{0, 1, 2, 3\}$,

(second) the inequality $x + 3 < 5$:

At $x = 0$, $0 + 3 = 3 < 5$, so "zero" verifies the inequality.

At $x = 1$, $1 + 3 = 4 < 5$, so "1" verifies the inequality.

At $x = 2$, $2 + 3 = 5 \nless 5$, so "2" does not verify the inequality.

At $x = 3$, $3 + 3 = 6 \nless 5$, so "3" does not verify the inequality.

We get the solution set = $\{0, 1\}$. Notice that $\{0, 1\} \subset \{0, 1, 2, 3\}$

From the pervious example, we can conclude :

- The first degree equation in one unknown has only one solution which is one of the elements of the substitution set.
- The first degree inequality in one unknown may have one value or more in its solution set and these are elements of the substitution set.

- The substitution set is the set which the unknown (symbol) in the equation or the inequality belongs to it.

- The solution set is the set of elements which verify the equation or the inequality.

Exercise (2 - 1)

(1) Determine the degree and the unknown :

(a) $x - 7 = 1$ (b) $x + 3 > 2$ (c) $2x^2 - 2 = 14$

(d) $x - 2y = 5$ (e) $3x - 2 < -2$ (f) $x^3 - 4x^2 = 0$

(2) Given that the substitution set is $M = \{-1, -2, 0, 2\}$

(a) Find the solution set of the equation $2x + 1 = 5$.

(b) Find the solution set of the inequality $x - 3 < -1$.

(3) Find the solution set of each of the following equations and inequalities :

(1) $x + 5 = 12$, if the substitution set is $\{3, 5, 7, 8\}$.

(2) $2x + 4 = 14$, if the substitution set is $\{-2, 2, 3, 5\}$.

(3) $4x - 3 = 9$, if the substitution set is $\{2, 3, 4\}$.

(4) $2(x - 3) = x + 1$, if the substitution set is $\{4, 5, 6, 7\}$.

(5) $x + 3 < 5$, if the substitution set is $\{4, 3, 2, 1, 0\}$.

(6) $3x - 1 > -2$, if the substitution set is $\{-2, -1, 0, 1, 2\}$.

(7) $-x + 1 < 4$, if the substitution set is $\{-3, -2, 0, 2, 3\}$.

(8) $2x + 5 > 2$, if the substitution set is $\{-3, -2, -1, 0, 1\}$.

2 Solving first degree equations in one unknown

What do you learn from this lesson ?
Through your active participation you will come to :
- properties of equality in N, Z.
- Adding and subtracting property in N, Z.
- Multiplication and division properties in N, Z.
- Solving first degree equations in one unknown using the properties of equality in N, Z.

Mathematical Concepts
- Addition and subtraction.
- Multiplication and division.

You know that :

Solving equation means getting the value of the unknown (symbol) in the equation. In the previous lesson, we used the substitution set to get the solution set. But this method is too long and difficult and may be impossible if the substitution set is infinite like N or Z.
So you will study easy and simple methods which depends mainly on the properties of equality in N, Z. these properties will be discussed as follows :

Properties of equality in N, Z

(1) Adding and subtracting property

- The figure (1) opposite expresses the equality of two balance pans where :
- The first pan has a sack having unknown number of apples plus 4 apples.
- The second pan has 7 apples. The balance can be expressed in this case by the equation : $x + 4 = 7$

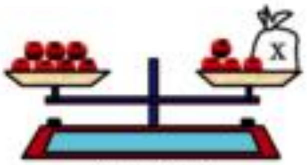


figure (1)

- If we add 2 apples to each pan (figure 2) then the two pans remain in state of equality and we express that case by the equation:

$x + 4 + 2 = 7 + 2$ i.e $x + 6 = 9$

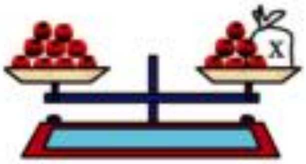


figure (2)

- If we cancel 6 apples from each pan (figure 3) then the two pans remain in state of equality as in figure (3) and this is expressed by the equation : $x + 6 - 6 = 9 - 6$ i.e $x = 3$

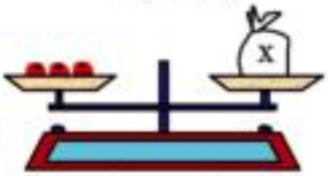


figure (3)

From the previous discussion, we deduce that :

If a, b, c are three Integers, and a = b, then

$$a + c = b + c$$

$$, a - c = b - c.$$

The question now is :

How can we use adding and subtracting properties in solving first degree equations in one unknown in N, Z ? - This will be clear through the following examples :

Example (1) :

Solve the equation $x - 2 = 3$ in Z

Solution :

$\therefore\ x - 2 = 3$ (by adding 2 to both sides)

$\therefore\ x - 2 + 2 = 3 + 2$ (additive inverse property)

$\therefore\ x + 0 = 5$ (Identity element)

$\therefore\ x = 5$, then the solution set = $\{5\}$.

or s.s. = $\{5\}$ where s.s. means solution set.

Check the solution

put $x = 5$ in the equation $x - 2 = 3$

$\therefore\ 5 - 2 = 3 \Longrightarrow 3 = 3$

(2) Multiplication and Division property :

- The figure (1) opposite expresses a state of equality between two balance pans.

- The first pan : has four pieces of metal of same weight and the weight of each is (x).

- The second pan : has two pieces of metal and each is of weight 40 gm. we can express the balance in this case by the equation.

$4x = 40 + 40$ i.e $4x = 80$.

La figure (1)

If we double the weight in both pans, then the first pan has (8) pieces of same weight (x) and the second pan has (4) pieces each of same weight 40 gm.

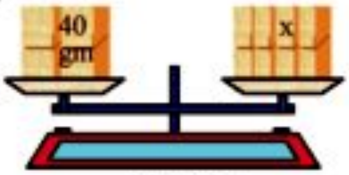


figure (2)

We can express the balance in this case by the equation :

$8x = 160$ which means $2 \times 4x = 2 \times 80$.

If we remove $\frac{3}{4}$ the weight of each pan, then the first pan has 2 pieces of the same weight (x) as in figure (3) and the second pan has one piece or weight 40 gm.

figure (3)

We can express the balance in this case by the equation :

$2x = 40$ i.e. $\frac{8x}{4} = \frac{160}{4}$

From the previous discussion, we deduce that :

> If a, b, c three integers, and a = b, then
>
> $$a \times c = b \times c$$
>
> $$, a \div c = b \div c , c \neq 0$$

The question now is :

How can we use the multiplication and division properties in solving first degree equations in one unknown in N, Z ?

- This will be clear through the following examples :

Example (2) :

Solve the equation $4x = 24$ in N.

Solution :

$\frac{4x}{4} = \frac{24}{4}$ by dividing both sides by 4

$\therefore x = 6$ i.e s.s = {6}

Example (3) :

Solve the equation : $\frac{x}{5} = 4$ in N

Solution :

$\frac{x}{5} \times 5 = 4 \times 5$ (multiplying both sides by 5)

$\therefore x = 20$ i.e s.s = {20}

Example (4) :

Solve the equation : $2x + 9 = -23$ in N and Z .

Solution :

First : **We start by applying adding and subtracting properties :**

$\because 2x + 9 = -23$ (by adding (-9) to both sides)

$\therefore 2x + 9 + (-9) = -23 + (-9)$ (Additive inverse property)

$\therefore 2x = -32$

Second : Applying multiplication and division properties

$\therefore 2x = -32$ (dividing both sides by 2)

$\therefore \frac{2x}{2} = \frac{-32}{2}$ i.e

$x = -16 \notin N$, then the equation has no solution in N and the s.s. = φ

, $x = -16 \in Z$ i.e s.s. = $\{-16\}$

Example (5) :

Number when added to triple output become 72 find the solution.

The Solution :

Let the number is x → three times of $x = 3x$

Then $x + 3x = 72$

$4x = 72$ (dividing both sides by 4)

$\frac{4x}{4} = \frac{72}{4}$

$x = 18$

the number is 18

Exercise (2 - 2)

(1) Find the solution set of each the following equation :

(a) $x + 3 = 3$

(b) $x - 2 = 1$

(c) $2x = 6$

(d) $2x + 1 = -5$

(e) $2x =$ zero

(2) Solve the following equations :

(a) $x + 3 = 9$ in N

(b) $x - 22 = 18$ in Z

(3) Find the solution set of each of the following equations in N :

(a) $x + 8 = 19$

(b) $4x + 1 = 17$

(c) $6x + 7 = 25$

(4) Find the solution set of each of the following equations Z :

(a) $x - 12 = 40$

(b) $3x - 2 = -19$

(5) Study the possibility of solving each of the following equations in N, Z :

(a) $3x = 8$

(b) $3M + 12 = 6$

(c) $2L - 15 = 8$

3

Solving first Degree Inequality in one unknown

What do you learn from this lesson?
Through your active participation you will come to :

- Properties of inequality in N, Z namely :
  - Adding and subtracting in N, Z.
  - Multiplying and dividing property in N, Z.
  - Solving first degree inequality in N, Z .

Mathematical Concepts

- Adding and subtracting.
- Multiplying and dividing.

From the previous lessons, you have learned the solution of the first degree equations using the properties of equality.

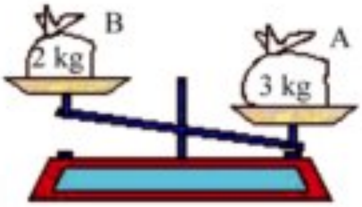
In this lesson, you learn solving first degree inequalities in one unknown using the properties of inequality in N, Z.

Properties of inequality in N, Z

(a) Adding and subtracting property

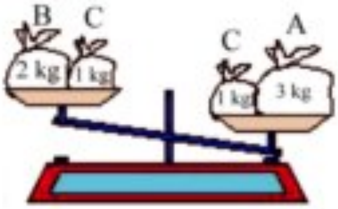
Notice and Discuss

Adding : The figure (1) opposite has two unequal balance pans.



The Figure (1)

- The first pan has a sack of rice of weight 3 kg.
- The other pan has another sack of rice of weight 2 kg.
- It is clear that the sack (A) is heavier than the sack (B) and this is expressed by the inequality ($3 > 2$) or ($a > b$).
- By adding another sack (c) of weight 1 kg to each of the two pans, we notice that the balance remains in its position as in figure (2).



The Figure (2)

We can express this case by the inequality ($3 + 1 > 2 + 1$) or ($a + c > b + c$).

Subtracting :

by removing the sack (c) from both pans figure (3), we notice that the balance goes back to the first case in figure (1).

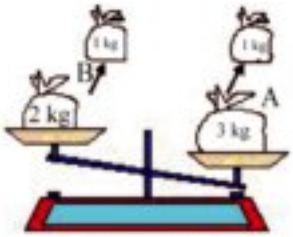


Figure (3)

from the previous discussion, we deduce :

If a, b, c are three integers and $a > b$, than $a + c > b + c$

(b) Multiplication and Division property

Multiplication :

figure (1) opposite represents two unequal balance pans.

In the first pan : A weight (A) of 2 kg.

In the second pan : A weight (B) of 1 kg.

In the other a weight (b) of 1 kg.

It is clear that we can express this position by the inequality ($a > b$).

If we doubled the weight in each pan, what do you expect ?

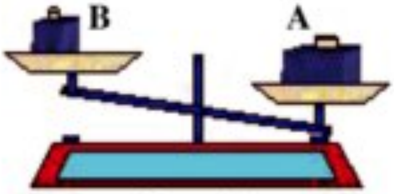


Figure (1)

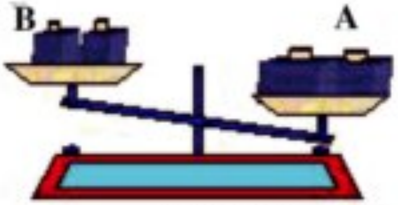


Figure (2)

Notice :

The balance will remain in its same position as in figure (2).

we can express this case by the inequality

($2 + 2 > 1 + 1$) i.e ($2 \times 2 > 2 \times 1$) which means ($2 \times a > 2 \times b$).

for example :

(1) It is known that $7 > 5$, then multiplying both sides by 3, we get

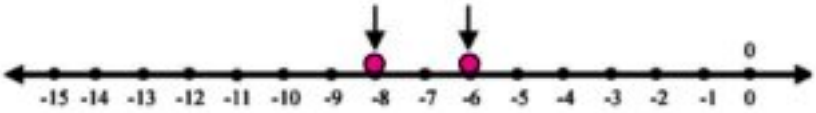
$3 \times 7 > 3 \times 5$ i.e $21 > 15$ (true relation)

(2) It is known that $4 > 3$, then multiplying both side by (-2), we get

$-2 \times 4 < -2 \times 3$ i.e $-8 < -6$ (true relation)

Notice :

The change in the inequality sign from (>) to (<) when multiplying by a negative number.
(-2) makes the number -8 left to the number -6 on the number line.

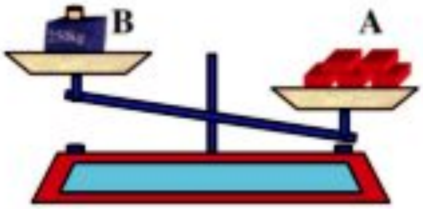


From the previous, we deduce the following :

If a, b, c are three integers, and

- If $a > b$, $c > 0$, then $ac > bc$
- If $a > b$, $c < 0$, then $ac < bc$

Division:



The Figure opposite illustrates the two pans of a balance as follows In the first pan: five chocolate pieces equal in weight and each is of weight (x).

In the second pan : A weight of magnitude 250 gm.

We can express the balance position by the inequality

$(5 \times > 250)$ i.e $(5 \times x > 5 \times 50)$

by dividing both side by 5, we get $x > 50$ (true relation)

Notice :

When dividing by a negative number the direction of the inequality sign changes.

e.g : if $-3x < 30$ i.e $-3 \times x < 3 \times 10$

Then dividing by -3, we get $x > -10$ (True relation)

From the previous, we can deduce the following

If a, b, c are three integers and

(i) $a c < b c$, $c > 0$, then $a < b$

(ii) $a c < b c$, $c < 0$, then $a > b$

Remarks :

we can summarize the four operations on inequalities in N, Z as follows :

(a) A constant number can be added to both sides of the inequality without changing its direction.

(b) A constant number can be subtracted from both sides of the inequality without changing its direction (so that the subtraction is possible).

(c) A constant number (positive) can be multiplied (divided) by both sides of the inequality without changing its direction.

(d) when multiplying (dividing) both sides of the inequality by a negative number, then its direction must be changed.

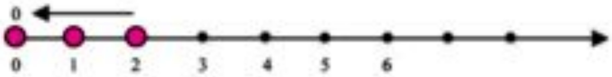
Example (1) :

Find the solution set of the inequality $x + 4 < 7$ where $x \in N$, then represent it on the number line.

Solution :

(1) In N $x + 4 < 7$ (subtracting 4 from both sides)

$\therefore x + 4 - 4 < 7 - 4$ i.e $x < 3$, then s.s = {0, 1, 2}



Example (2)

Find the solution set

of the inequality $2x + 9 < 1$ and represent it on the number line if

(a) $x \in N$ (b) $x \in Z$

Solution:

(1) In N : $2x + 9 < 1$ (subtracting 9)

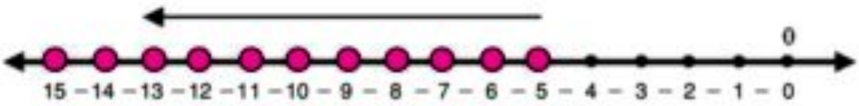
$\therefore 2x + 9 - 9 < 1 - 9$ i.e $2x < -8$ (dividing by 2)

$\therefore x < (-4)$ (not possible in N)

$\therefore$ s.s. = φ

(2) In Z $\because x < -4$ (possible in Z)

$\therefore$ s.s. = {-5 , -6 , -7 , ………………….}



Example (3) :

Find the solution set of the Inequality $3x - 2 \geq 4$ where $x \in \mathbb{Z}$,

Then represent it on the number line.

Solution :

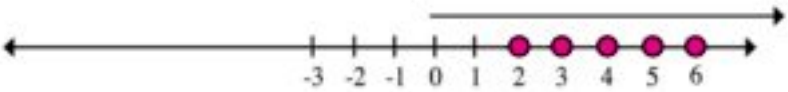
$3x - 2 + 2 \geq 4 + 2$ (additive inverse property)

$3x + 0 \geq 6$ (additive Identity property)

$3x \geq 6$ (dividing by 3)

$x \geq 2$

$\therefore$ S.s = {2, 3, 4, 5, ... }



Exercise (2 - 3)

First :

Express Symbolically each of the following :

(1) x is less than -5

(2) x is great than or equal to 3

(3) x is less than or equal to 2

(4) x is less than 5 and orent than 2

(5) x is less than or equal to 7 and greater than 1

(6) x is less than or equal to 1 and greater than or equal to (-4)

Secand :

Complet when $x \in Z$

(1) If $x + 5 > 2$ then $x >$ ....

(2) If $2x + 1 \geq 5$ then $2x \geq$ .... , $x \geq$ ....

(3) If $3x - 1 \leq 8$ then $3x \leq$ .... , $x \leq$ ....

Third :

Find the solution set each of the following inequalities represent the solution set on the number line:

(1) $x - 3 < 1$, Where $x \in N$

(2) $2x - 5 \leq -7$, Where $x \in Z$

(3) $3x + 2 \leq 11$, Where $x \in N$

(4) $3x - 7 \leq 5$, Where $x \in Z$

(5) $2x - 3 \geq 1$, Where $x \in Z$

General Exercises on unit (2)

(1) Which of the following represents an equation ? and Why ?

(a) $x - 21$ (b) $10 - 12 = -2$ (c) $2x - 3 = 5$

(2) Which of the following represents an inequality or an equation ? Give reason :

(a) $x > 7 - 5$ (b) $3x + 2 = 11$

(c) $x < -35$ (d) $2x = 24$

(3) Determine the degree of each of the following equations :

(a) $3x - 9 = 2$ (b) $3x^2 - 6 = 14$

(4) Given that the substitution set is $M = \{0, 1, 2, 3\}$

(a) Find the solution set of the equation : $2x - 7 = -1$

(b) Find the solution set of the inequality : $x + 4 > 5$

(5) (i) **S**olve in N :

(a) $x + 7 = 22$ (b) $8x = 32$

(c) $4x + 3 = 23$

(ii) **S**olve in Z :

(a) $x - 12 = 6$ (b) $\frac{x - 3}{4} = -2$

(c) $3 - 2x = 9$

(6) (i) **S**olve in N :

(a) $x + 3 < 7$ (b) $2x + 1 \leq 5$

(ii) **S**olve in Z :

(a) $1 - 8x < 33$ (b) $2x - 3 < 5$

Technological Activity :

Finding the solution of the first degree equation in one unknown using Excel program.

What do you learn from this activity, using Excel in :

*** Input a set of Integers through Excel program.**

*** Finding the solution of the first degree equation in one unknown.**

Example :

Find the solution set of the equation :

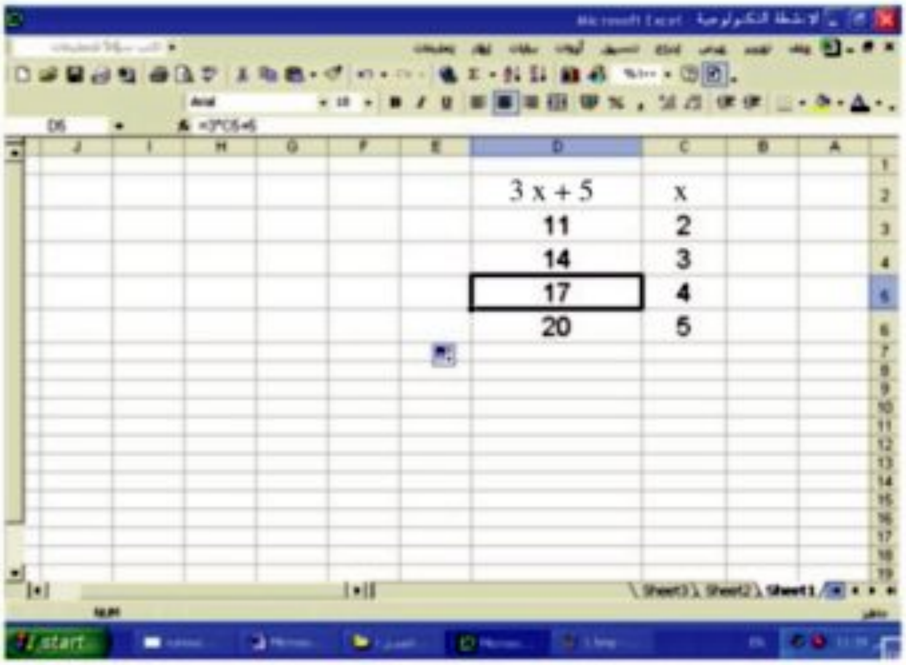
$3x + 5 = 17$, if the substitution set is $L = \{2, 3, 4, 5\}$

Steps :

1 - Press "START" ® Program ® Microsoft Excel.

2 - Write the elements of the substituting set in the cells, below x in Excel Sheet.

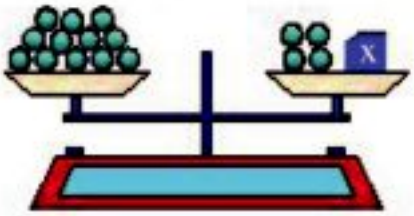
3 - To find the value of x which verifies the equation, Mark the cell D_3 and write in it $= 3*C_3 + 5$, then press the key (Enter) to the result (11) and through marking the cell D_3 and dragging down wards from the left lower corner to the end of the rows, the results will apper as in the following figure.



3 x + 5	x
11	2
14	3
17	4
20	5

4 - From the data on the screen it is clear that $x = 4$ verifies the equation. i.e the s.s $= \{4\}$.

Below each balance, express the suitable mathematical statement, then solve it.

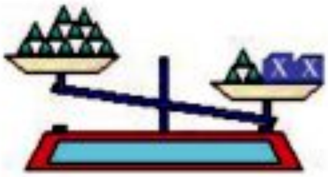


- The mathematical statement

……………………………………………

- Solution

……………………………………………

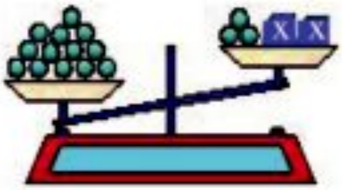


- The mathematical statement

……………………………………………

- Solution

……………………………………………

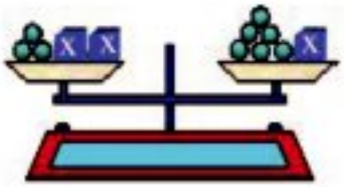


- The mathematical statement

……………………………………………

- Solution

……………………………………………



- The mathematical statement

……………………………………………

- Solution

……………………………………………

The Unit Test

(1) Complete each of the following :

(a) The equation is a mathematical sentence ……………

(b) The inequality is a mathematical sentence ……………

(c) The substitution set is ……………………………………

(d) The solution set is ……………………………………….

(2) Choose the answer which verifies the equations and the inequalities from those between brackets:

(a) $3x + 1 = -5$	$\{0, -1, 1, -2\}$
(b) $x - 1 = -2$	$\{3, 0, -1, 1\}$
(c) $x - 2 > 3$	$\{3, 4, 5, 6\}$
d) $2x + 1 \leq -1$	$\{4, 2, 0, -1\}$

(3) Solve the following inequalities and represent the solution set on the number line :

(a) $3x + 2 \geq 11$, where $x \in N$

(b) $4x + 1 < 13$, where $x \in Z$

(4) Solve the following equations in Z :

(a) $6x - 2 = 14$

(b) $2x + 1 = -9$

(c) $7x + 5 = 26$

(d) $4 - 2x = 24$

Unit three

Geometry and measurement

Lesson (1) : Distance between two points in the coordinate plane

Lesson (2) : Geometric transformation: Translation

Lesson (3) : Area of the circle

Lesson (4) : Lateral area and the total area of a : (cube - cuboid)

- ***General exercises on the unit***
- ***Technological activity***
- ***Activity of the unit***
- ***Unit test***

1 *Distance between two points in the coordinates plane*

What do you learn from this lesson?

From your active participation you can come to :

1- Calculate the distance between two points on a ray.

2- Calculate the distance between two points in the coordinate plane N.

3- Calculate the distance between two points on a straight line.

4- Calculate the distance between two points in the coordinate plane Z .

5- Determining points in the coordinate plane Z.

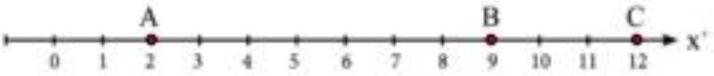
The mathematical concepts.

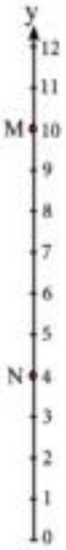
- Horizontal line.
- Vertical line.
- Coordinate plane Z.

Think and discuss :

a) The distance between two points on a ray :

You have studied before the distance between two points on a horizontal ray or vertical ray. Notice from the opposite figure :





The points A, B and C represent the numbers 2 , 9 and 12 respectively.

The distance between the two points A and B is :

Length of $\overline{AB}$ = coordinate of the ending point – coordinate of the starting point.

= 9 – 2 = 7 cm.

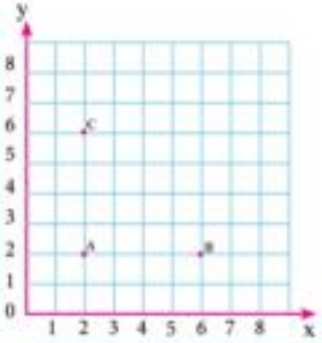
Complete :

AC = ……. - …….. = …….. cm.

BC = …….. - …….. = …….. cm.

NM = …….. - …….. = …….. cm.

b) Distance between two points in the coordinates plane for the natural numbers N.



You have studied before the coordinate plane for the natural numbers N , which is the union of the horizontal ray $\overrightarrow{OX}$ and the vertical ray $\overrightarrow{OY}$. As in the opposite figure :

The position of any point in the coordinate plane for the natural numbers is determined by a unique ordered pair.

Notice : From the opposite figure :

A (2 , 2) , B (6 , 2) , and C (2 , 6)

2- Determine "is it parallel to $\overrightarrow{OX}$ or $\overrightarrow{OY}$?"

3- If it is parallel to $\overrightarrow{OX}$, calculate the distance as on a horizontal ray and if it is parallel to $\overrightarrow{OY}$, calculate the distance as on a vertical ray.

Complete from the previous figure :

AB = ……………… units.

AC = ……………… units.

The type of the triangle Δ ABC with respect to its sides is ………..

c) Calculate the distance between two points on a straight line.

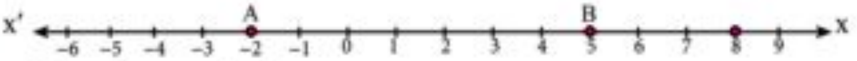
- The straight line which mentioned here is the integers number line whether horizontal or vertical. As you know it is an enlargement of the ray of the natural numbers by adding Y.

- When calculating the distance between two points on the integers number line we care about :

1- The absolute value which is

$$= \left|\, \textit{number of the ending point} - \textit{number of the starting point} \,\right|$$

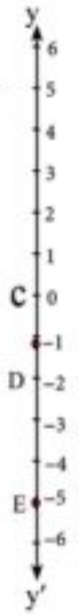
2- The properties of addition and subtraction in Z.



Notice : From the opposite figure :

The point A represents the number (-2), the point B represents the number (5), then

$AB = |B - A| = |5 - (-2)| = |5 + 2| = 7$ *units.*



Complete :

$DC = |\,\ldots\ldots\,| = |\,\ldots\ldots\,| = |\,\ldots\ldots\,| = \ldots\ldots$ *units.*

$DE = \ldots\ldots\ldots = \ldots\ldots\ldots = \ldots\ldots\ldots$ *units.*

d) Calculate the distance between two points in the coordinate plane Z.

The opposite figure represents the plane of the integer numbers :

Notice : The position of any point in the coordinate plane for the integer numbers is determined by an ordered pair (x , y) .

- Calculating the distance between two points in the coordinate plane Z is as the same as what happens in the N plane in addition to considering :

- The enlargement and extension of the numbers by adding Y^-.

- The properties of addition and subtraction in Z.

From the figure :

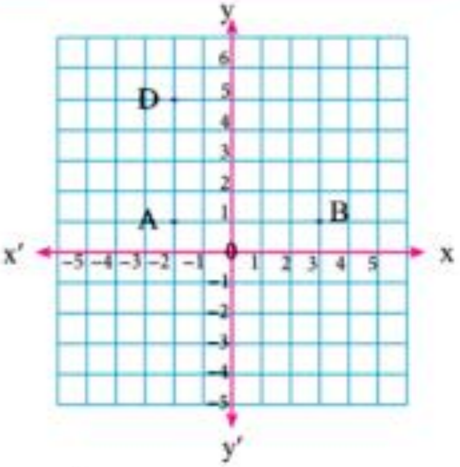
A (-2 , 1), B (3 , 1) and D (-2 , 5)

$\overline{AB} \parallel \overleftrightarrow{xx'}$; $\overline{AD} \parallel \overleftrightarrow{yy'}$

AB = | B – A| = | 3 – (-2)| = 5 cm

AD = ………… = ………… = …………cm.

- Determine the position of the point C (3 , 5), then satisfy that the shape ABCD is a parallelogram, Calculate its perimeter and its area.



Example :

In the opposite Co-ordinates plane

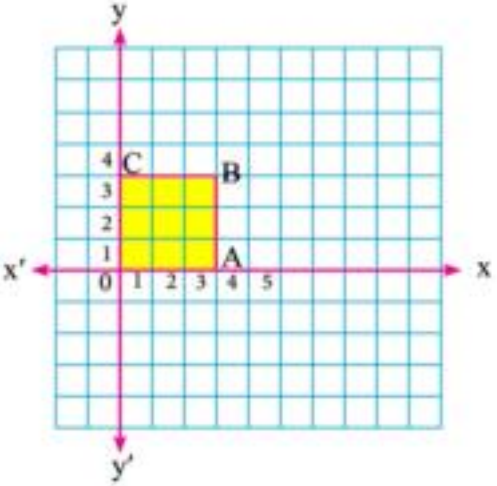
If OABC is a square, Where O (0,0) ,

A (3,0) , B (x , y) , C (0,3)

(a) Determine the position of each of the following paints O , A , C

(b) Find the ordered pair (x , y) that represent the verte & B

(c) Calculate area, perimeter of square OABC



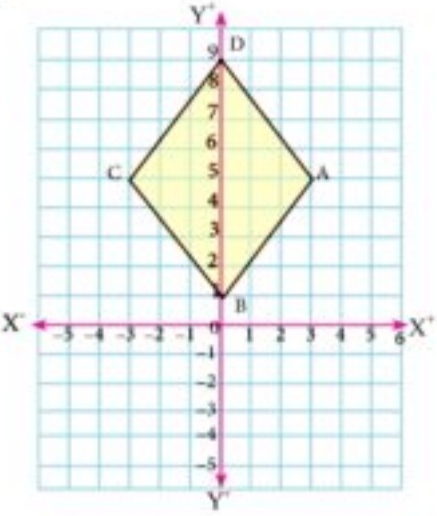
Solution :

(a) The points O, A , C are determined as in the figure.

(b) Since OABC is a square, then B (x , y) = (3,3).

(c) Perimeter = 4 × side length = 4 × 3 = 12 unit length

Exercise (3 - 1)



(1) In the opposite coordinate plane : ABCD is a rhombus.

(a) Complete the coordinates of the following points :

A (…… , ……) , B (…… , ……)

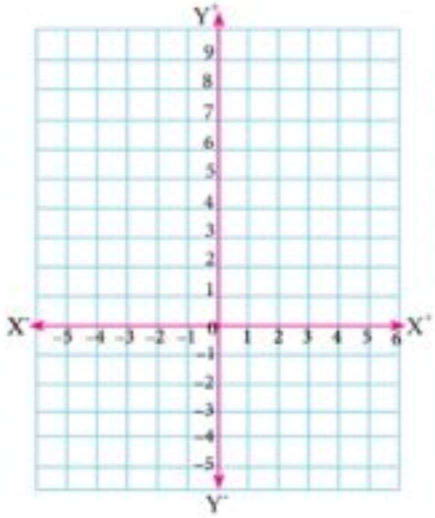
C (…… , ……) , D (…… , ……)

(b) The area of the rhombus ABCD can be calculated by using the length of its perpendicular diagonals, where :

The length of $\overline{AC}$ = …………,

The length of $\overline{BD}$ = …………

Surface area of the rhombus = …………



(2) In the opposite coordinate plane :

(a) Determine the position of the following points :

L (-1 , 1) , M (1 , 1)

N (1 , 8) , E (-1 , 8)

(b) Find the perimeter and the area of the shape LMNH.

(c) Determine whether the shape is symmetric about y.axis or not ? Why ?

2

Geometric transformations. Translation transformation

What do you learn from this lesson?

From your active participation you can come to:

1 - The concept of the geometric transformation, the concept of translation.

2 - Find the image of a point by translation in the page plane.

3 - Find the image of a point by translation in the coordinate plane.

4 - Find the image of a line segment by translation in the coordinate plan.

5 - Find the image of a geometric shape by translation in the coordinate plane.

6 - Determine the symmetry, reflection and translation through life examples.

The mathematical concepts.

The geometric transformation.
Translation.
The page plane.
The coordinate plane.

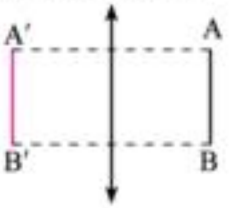
Think and discuss.

You have studied before the geometric transformation and you have known that :

Geometric transformation transforms each point in the plane in to a point A′in the same plane.
Also, you studied the reflection transformation :

In the opposite figure $\overline{A'B'}$ is the image of $\overline{AB}$ by reflection in L.

L is the axis of reflection.



$A'B' = AB$

$\overline{A'B'} \parallel \overline{AB}$

What is the name of the figure ABA´B´? Why?

Are there axes of symmetry for the shape ?

State they if there exist.

Now, we will study the translation.

In the opposite figure Hany wants to pass the ball to Ahmed.

In order to translate the ball to Ahmed, two things must be known which are :

- The ball moves all the distance from Hany to Ahmed.
- The ball goes to the direction of Ahmed.

That means : Two things must be known for the translation to happen,

* magnitude of the translation.
* direction of the translation.

In the picture :

-Magnitude of the translation (the distance between Hany to Ahmed).

-Direction of the translation (the direction from Hany to Ahmed).

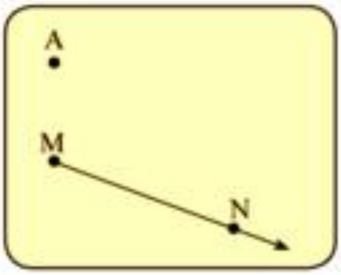
We will study the following cases of translation which are:

First : Translation of a point in the plane.

(a) In the page plane.

Work and discuss.

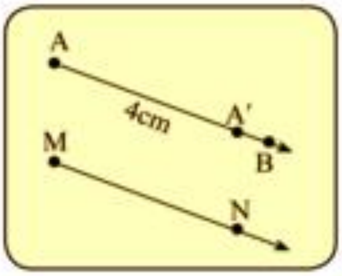
Activity (1) : Through the page plane.



Draw MN, determine the point $A \notin \overrightarrow{MN}$ as the figure :

Required : translate the point A by distance 4cm in the direction of $\overrightarrow{MN}$.

Solution :



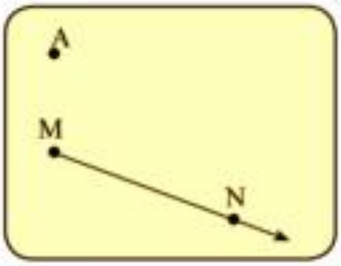
Draw a ray from A parallel to $\overrightarrow{MN}$ to take its direction as AB in the opposite figure .

Determine the point A′ on $\overrightarrow{AB}$ such that AA′ = 4cm.

> **Notice :** A′ is the image of A by translation its magnitude is 4cm in direction of $\overrightarrow{MN}$.

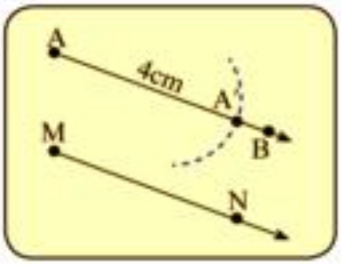
In the previous example : The magnitude of translation is 4cm,

The direction of translation is $\overrightarrow{MN}$.



Activity (2) : What happens if the required is : finding the image of the point A by translation MN in the direction of $\overrightarrow{MN}$.

Solution :



In the case, the magnitude of translation $\overrightarrow{MN}$ is known but not determined. Thus, we use the compass through the following steps :

Draw the ray $\overrightarrow{AB}$ from A parallel to $\overrightarrow{MN}$ and takes its direction.

Fix the pin of the compass at A then draw an arc of a circle its radius is $\overrightarrow{MN}$.

The point of intersection of the arc with $\overrightarrow{AB}$ is A′.

> **Notice :** A′ is the image of A by translation its magnitude is (MN) in the direction of $\overrightarrow{MN}$.
>
> $AA' = MN$, $\overline{AA'} // \overline{MN}$

(b) In the coordinate plane for integer numbers.

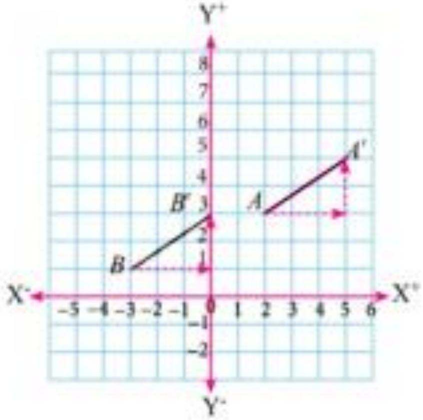
The translation in the coordinate plane transforms each point A in the plane to A′ in the same plane by displacement (c) in direction of the x – axis followed by another displacement (d) in direction of the y – axis,

Such that : $A\ (x\ ,\ y) \xrightarrow{yields} A'\ (x + c\ ,\ y + d)$

Example (1) :

In the opposite figure, find the image of the two points

A (3 , 2) , B (-3 , 1) by translation $(x + 3\ ,\ y + 2)$



Solution :

First : Determine the magnitude and direction of translation by displacement 3cm in the direction of x⁺ followed by displacement 2cm in the direction of y⁺.

Second : Find the image of each point one by one as follow :

$A' = (2 + 3\ ,\ \ 3 + 2) = (5\ ,\ 5)$

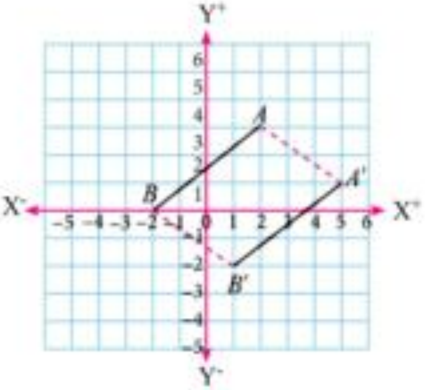
$B' = (-3 + 3\ ,\ 1 + 2) = (0\ ,\ 3)$

Notice : The point and the arrows on the drawing show the consecution of translation, magnitude and direction in each case.

Second : translation of a line segment in the plane.

Example (2) :

In the opposite figure, Find the image of the line segment $\overline{AB}$ where : A (2 , 3) , B (-2 , 0) by translation (x + 3 , y – 2).



Solution :

First : Determine the magnitude and direction of translation which are : displacement 3 cm in the direction of X⁺ followed by displacement 2cm in the direction of Y⁻ .

Second : Find the image of each point one by one as follow :

$A' = (2 + 3\ ,\ 3 - 2) = (5\ ,\ 1)$

$B' = (-2 + 3\ ,\ 0 - 2) = (1\ ,\ -2)$

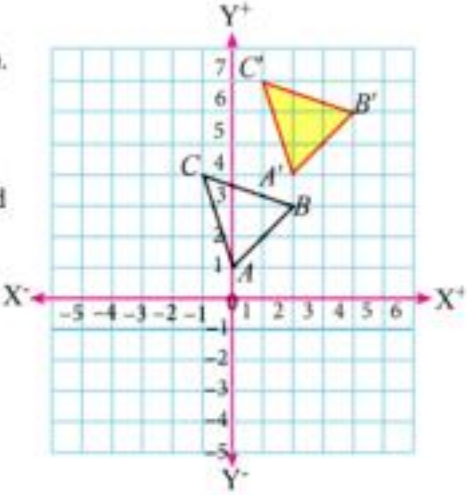
Notice : $\overline{A'B'}$ **is the image of** $\overline{AB}$ **by translation (x + 3 , y – 2).A′B′ = AB ,**

$\overline{A'B'}\ //\ \overline{AB}$

Third : Translation of a geometric shape in the coordinate plane

Example (3) :

In the opposite figure, Find the image of the Δ ABC where : A (0 , 1) , B (2 , 3) and C (-1 , 4) by translation (x + 2 , y + 3).



Solution :

First : Determine the magnitude and direction of translation which are: displacement 2 cm in the direction of X^+ followed by displacement 3 cm in the direction of y^+.

Second : Find the image of each point one by one as follow:

$A' = (0 + 2 , 1 + 3) = (2 , 4)$

$B' = (2 + 2 , 3 + 3) = (4 , 6)$

$C' = (-1 + 2 , 4 + 3) = (1 , 7)$

Third: Determine the points A′, B′ and C′in the plane, then join between them. We found Δ A′ B′C′ is the image of Δ ABC by translation (x + 2 , y + 3).

Notice: $\overline{A'B'}$ is the image of $\overline{AB}$ by translation (x + 2 , y + 3).
$\overline{A'B'} \parallel \overline{AB}$, $\overline{B'C'} \parallel \overline{BC}$ and $\overline{C'A'} \parallel \overline{CA}$

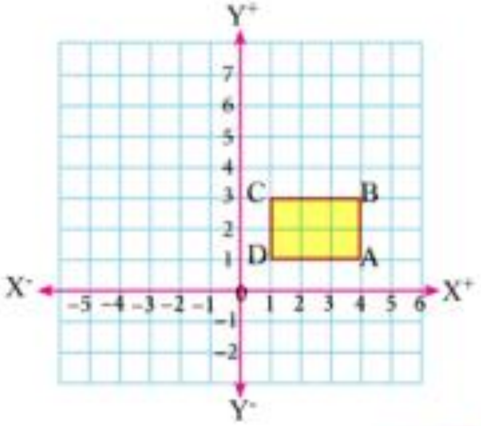
Drill (1) :

From the previous figure, complete :

(1) A′B′ = …………
(2) AC = …………
(3) $\overline{A'C'}$ // …………
(4) m(∠ B′) = m(∠……)
(5) m(∠ C) = m (∠……)
(6) $\overline{BC}$ // ……

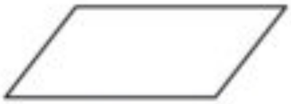
Drill (2) :

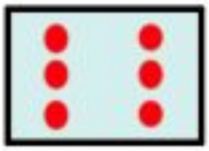
In the coordinate plane ABCD is a rectangle where : A (4 , 1) , B (4 , 3) , C (1 , 3) and D (1 , 1), Find its image by translation (x + 3 , y + 3)

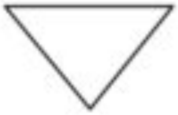


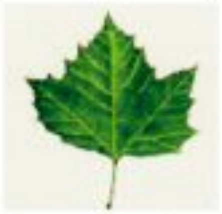
Exercise (3 – 2)

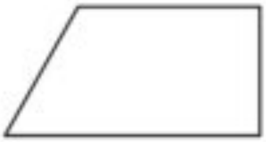
(1) Determine which of the following shapes is symmetric and which is not symmetric, and then draw the axes of symmetry.



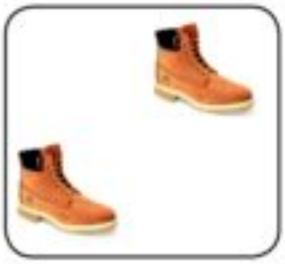


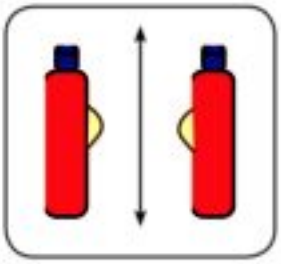


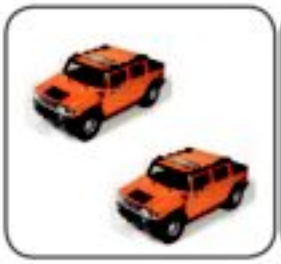


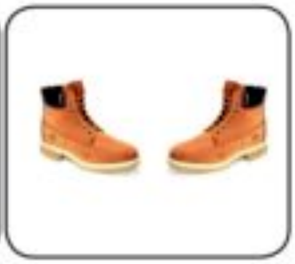


(2) In the following pictures determine the type of the geometric transformation (Reflection or Translation) and draw the direction of translation.





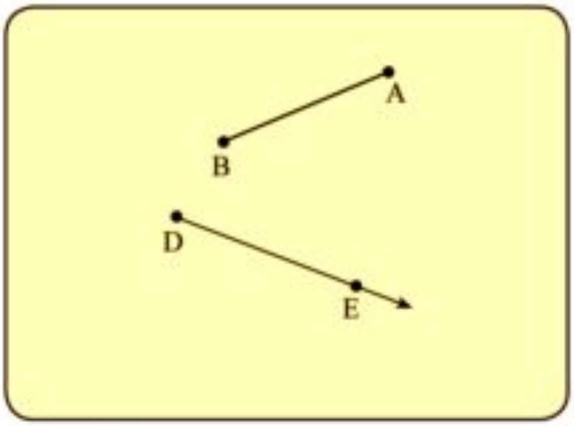


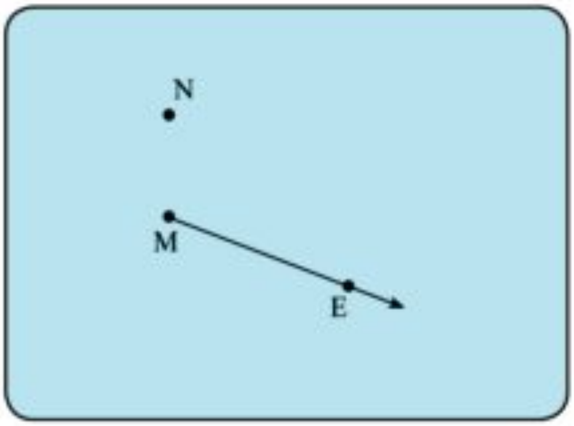


(3) Find the following :

(a) The image of the point N by translation $\overline{ME}$ in the direction of $\overrightarrow{ME}$.

(b) The image of the $\overline{AB}$ by translation its magnitude is 3 cm in the direction of $\overrightarrow{DE}$.





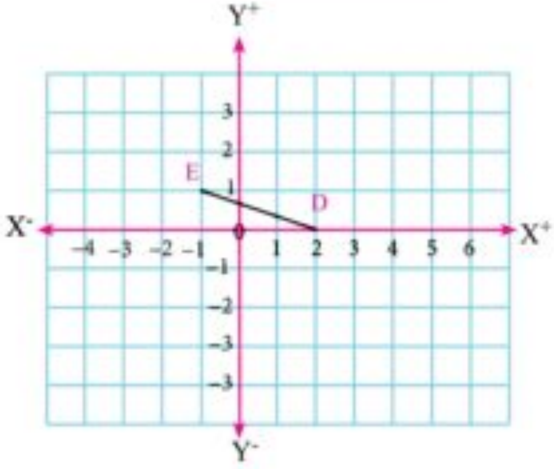
(4) In the opposite coordinate plane :

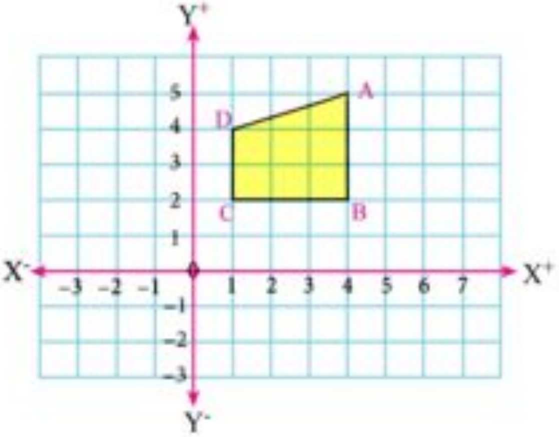
Determine the following :

a) The image of $\overline{DE}$ where D (2 , 0)

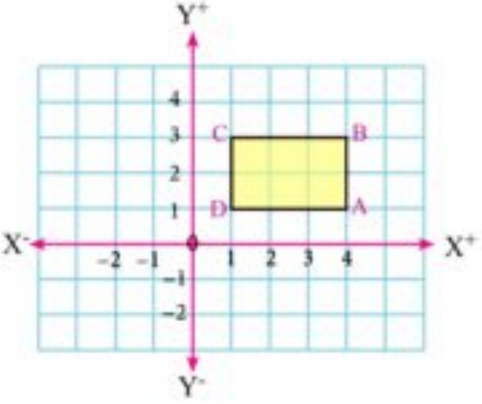
E (-1 , 1) by translation (x + 3 , y + 2)

What is name of the shape DD′E′E? Why ?





b) The image of the quadrilateral ABCD by translation (3 , -4).



c) On a square Lettice, draw the rectangle ABCD where: A (4,1) , B (4,3) , C (1,3), D (1,1) then find its image by the translation (x + 1, y + 2).

(5) Choose the correct answer :

a) The image of the point (-1,2) by translation of magnitude of 3 units in the positive direction of the x-axis is ....

[(-1,5) , (2,2) , (-2,2) , (-1,3)]

b) The image of the point (-3,4) by translation of magnitude of 4 units in the negative direction of the y-axis is ....

[(-3,0) , (-7,4) , (-3,8) , (-1,4)]

c) The image of the point (3,5) by translation (x + 2 , y - 1) is ....

[(5,6) , (5,4) , (1,4) , (1,6)]

d) The image of the point (......... , ...) by translation (x -3 , y + 4) is (-5, -3)

[(- 8,15) , (-2,7) , (-8,7) , (-2,-7)]

e) The image of the point (8, -10) by translation (-3,4) is ....

[(5, -6) , (5, -14) , (11, -6) , (11, -14)]

h The image of the point (1, -3) by translation (......... , ...) is (1,0)

[(1, 0) , (0, 0) , (3, 0) , (0, 3)]

3 *Area of the circle*

What do you learn from this lesson ?

From your active participation you can come to:

1 - The definition of the circular sector.

2 - Dividing the surface of the circle into equal sectors.

3 - Deducing a rule to calculate the surface area of the circle using a practical method depends on the circular sectors.

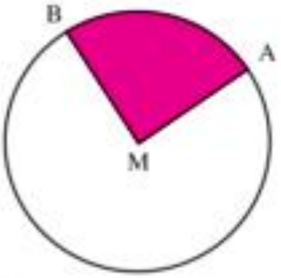
4 - Solve different applications on the surface area of the circle.

The mathematical concepts.

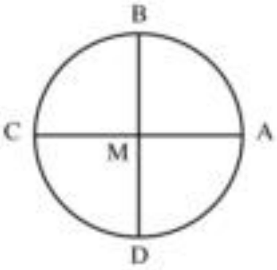
The circular sector.

Notice and discuss.

You have studied before "the circular sector" in the opposite figure the shaded part represents the circular sector (MAB) or (AMB).



The circular sector : is a part of the surface of the circle bounded by an arc and two radii passing through ends of the arc.



Activity (1) :

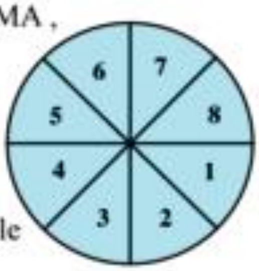
In the opposite figure, M is a circle and $\overline{AC}$, $\overline{BD}$ are two diameters, $\overline{MA}$, $\overline{MB}$, $\overline{MC}$, $\overline{MD}$ are radii.

Think carefully, then complete the following :

The surface of the circle M is divided into …….. circular sectors.

The relation between the resulted circular sectors is …………

The ratio between any circular sector to the surface of the circle is …………..



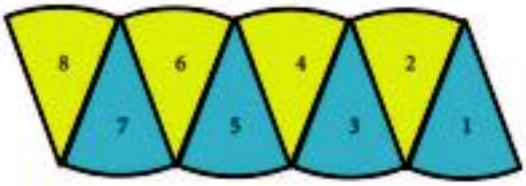
Activity (2) :

Draw the previous circle, then divided it into 8 equal circular sectors, by drawing another two diameters, be careful to the equality of length of the resulted arcs, number the resulted circular sectors from 1 to 8 as the opposite figure.

Draw the same circle using cardboard by the same numbering of the eight circular sectors.

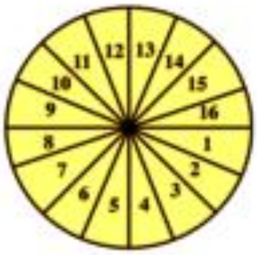
First, cut the circle then cut the resulted eight circular sectors one by one.

Using the glue, arrange the resulted circular sectors on the page of paper, such that the circular sectors which has the odd numbers its vertex up, and the circular sectors which has the even numbers its vertex down to have the opposite figure.

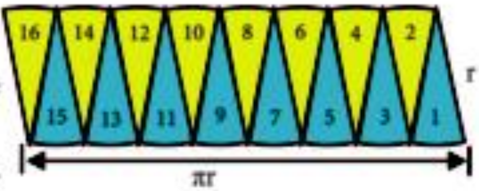


You may notice that the resulted shape formed from arranging the circular sectors is closer to be rectangle.

Draw the previous circle M, by its 8 equal circular sectors using cardboard, then divide it again into 16 equal circular sectors, by drawing another diameter between each two diameters to have 8 diameters and 16 equal circular sectors numbering from 1 to 16 as the opposite figure.



First, cut the circular sectors and use the glue to arrange resulted circular sectors on the page of paper by the same previous method to have the opposite figure.



You can notice that :

The resulted shape is closer to be rectangle more than the previous rectangle.

If the number of the circular sectors is increasing, then the shape will be closer to rectangle.

The length of the resulted rectangle = half of the circumference of the circle = πr.

The width of the resulted rectangle = radius of the circle = r.

That means : the surface area of the circle = the area of the resulted rectangle.

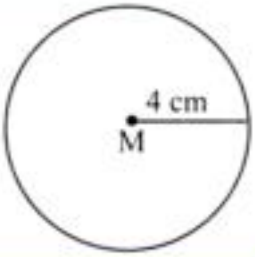
= length x width

$= \pi r \times r = \pi r^2$

The surface area of the circle $= \pi r^2$

Example (1) :

In the opposite figure, calculate the surface area of the circle M



Solution :

The surface area of the circle $= \pi r^2$

$= 3.14 \times 4 \times 4 = 50.24 \text{ cm}^2$

Notice that: π (as you studied before) is the approximately ratio between the circumference and the diameter of the circle it is $\simeq \frac{22}{7}$ or 3.14 and r is the abbreviation of the length of radius which represents its length.

You can use the calculator to carry out the approximation to find the required solution.

Example (2) :

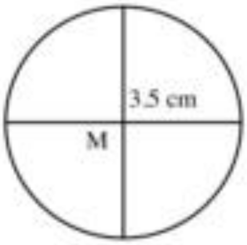
A circle its diameter is 14 cm, calculate its surface area where $\pi \simeq \frac{22}{7}$.

Solution :

The surface area of the circle $\pi r^2 = \frac{22}{7} \times 7 \times 7 = 154 \text{ cm}^2$

Example (3) :

In the opposite figure, a circle M of radius 3.5 cm, is divided into four equal circular sectors. Calculate the surface area of one sector where $\pi \simeq \frac{22}{7}$.



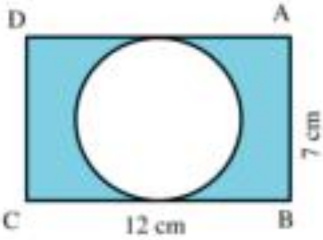
Solution :

The surface area of the circle $= \pi r^2 = \frac{22}{7} \times \frac{35}{10} \times \frac{35}{10} = 38.5 \text{ cm}^2$

The surface area of one sector $= \frac{38.5}{4} \simeq 9.625 \text{ cm}^2$

Example (4) :

In the opposite figure, ABCD is a rectangle its length 12 cm, its width 7 cm . A circle is drawn to touch the sides $\overline{AD}$ and $\overline{BC}$. Calculate the area of the shaded part where ($\pi \simeq \frac{22}{7}$)



Solution :

The area of the rectangle = $12 \times 7 = 84 \text{ cm}^2$

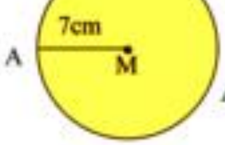
The area of the circle $= \pi r^2 = \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} = 38.5 \text{ cm}^2$

The area of the shaded part $= 84 - 38.5 = 45.5 \text{ cm}^2$

Exercise (3 – 3)

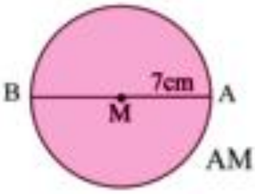
(1) Find the area of each of the following where $\pi = \frac{22}{7}$

(a)



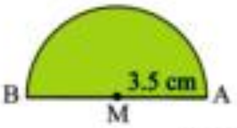
AM = 7cm

(b)



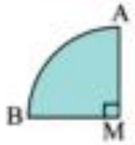
AM = 21 cm

(c)



AM = 3.5 cm

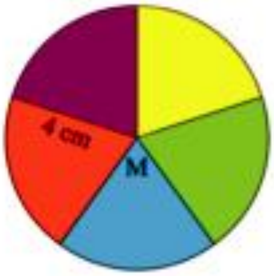
(d)



AM = 7cm

(2) A circle its diameter is 12 cm, calculate its surface area where ($\pi = \frac{22}{7}$ or 3.14)

(3) In the opposite figure, a circle M of radius 4 cm, is divided into five equal circular sectors. Calculate the surface area of one sector where ($\pi = 3.14$ or $\frac{22}{7}$)

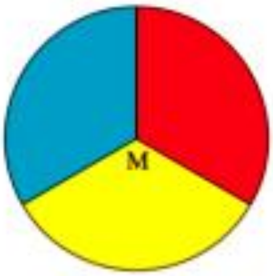


(4) A circle its circumference is 62.8 cm. Calculate its surface area. ($\pi = \frac{22}{7}$)

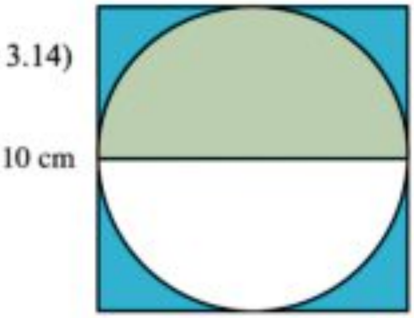
(5) In the opposite figure, a circle M is divided into three equal circular sectors, if the length of the arc of the sector is 44cm, and the perimeter of one sector is 86 cm, calculate :

(a) The radius of the circle

(b) The area of one sector ($\pi = \frac{22}{7}$).



(6) In the opposite figure, circle M is drawn inside the square of side length 10 cm to touch its sides.
Calculate the area of the shaded part where ($\pi = \frac{22}{7}$ or 3.14)



(7) A table its surface in the form of a circle, its diameter is 1.5m. Its surface is wanted to be covered by sheet of glass equals to its surface. Calculate the cost price if the square meter of the glass costs LE 60 ($\pi = \frac{22}{7}$ or 3.14)

(8) A circle its circumference is 44 cm. Calculate its surface area. ($\pi = \frac{22}{7}$ or 3.14)

4 *Lateral area and the total area For each of cube and cuboid*

What do you learn from this lesson ?

From your active participation you can come to :

1 - Calculate the lateral area of the cube.
2 - Calculate the total area of the cube.
3 - Calculate the lateral area of the cuboid.
4 - Calculate the total area of the cuboid.
5 - Solve miscellaneous exercises related to the lateral area and the total area for each of cube and cuboid.

The mathematical concepts.

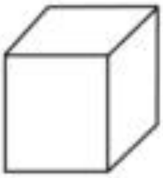
The lateral area.

The total area.

First : The cube :

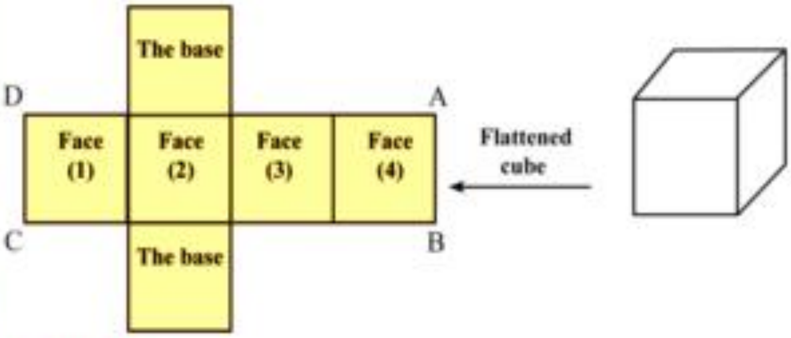
Notice and think.

You have studied before that the cube has six faces of equal squared shape also, it has 12 equal edges.



The lateral area of the cube.

Consider a carton box in the form of cube; flat the faces of the cube horizontally as the opposite figure.



Notice that : the faces 1, 2, 3, and 4 are the lateral faces and the lateral area is the sum of area of these faces.

The lateral area of the cube = Area of one face × 4

In another way : notice : When the faces of the cube were flattened, the rectangle ABCD was formed from the lateral faces.

Then, the length of the rectangle = the sum of the edge length of the four lateral faces 1, 2, 3 and 4

The width of the rectangle = the edge length $\overline{AB}$ which is the height of the cube.

The lateral area of the cube = Perimeter of the base × height

The total area of the cube.

In this case we take the area of the two bases in addition to the lateral area.

The total area of the cube = Area of one face × 6

Example (1) :

A cube of edge length 6 cm, Find its lateral area and its total area.

Solution :

The lateral area of the cube = Area of one face x 4

$= (6 \times 6) \times 4 = 36 \times 4 = 144 \text{ cm}^2$

In another way : The lateral area of the cube = Perimeter of the base x height

$= (6 \times 4) \times 6 = 24 \times 6 = 144 \text{ cm}^2$

The total area of the cube = Area of one face x 6

$= (6 \times 6) \times 6 = 36 \times 6 = 216 \text{ cm}^2$

Example (2) :

The total area of a cube is 486 cm^2 . Find the area of one face and its lateral area.

Solution :

The total area of the cube = Area of one face x 6

486 = Area of one face x 6

Area of one face = $\frac{486}{6} = 81 \text{ cm}^2$

The lateral area of the cube = Area of one face x 4

$= 81 \times 4 = 324 \text{ cm}^2$

Drill (1) :

The sum of edge lengths of a cube is 84 cm . Find its lateral area and its total area.

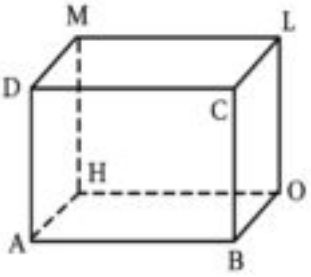
Second : The cuboid :

Notice and discuss :

You have studied before that the cuboid has six faces of rectangular shape; each two opposite faces are equal in area and parallel.

Activity :

Consider a carton box in the form of cuboid; its height h, find its lateral area, its total area.



The lateral area of the cuboid = the sum of areas of its four lateral faces 1, 2, 3 and 4.

They are rectangles perpendicular to the base, the width of each one = the height of the cuboid "H"

Then, the lateral area of the cuboid = AB x H + OB x H +

=OF x H + FA x H

= (AB + OB + OF + FA) x H

= Perimeter of the base x height

We deduce that :

- The lateral area of the cuboid = Perimeter of the base × height
- The total area of the cuboid = The lateral area + the sum of Areas of the two bases.

Example (3) :

A cuboid its length is 6 cm, its width is 4 cm, and its height is 8 cm.

Find its lateral area and its total area.

Solution :

The lateral area of the cuboid = Perimeter of the base x height

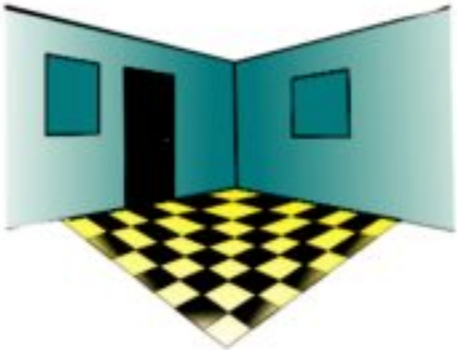
= (length + width) x 2 x height

= (6 + 4) x 2 x 8 = 20 x 8 = 160 cm^2

The total area of the cuboid = The lateral area + the sum of Areas of the two bases

= 160 + 2 x (6 x 4) = 160 + 48 = 208 cm^2

Example (4) :



A room in the form of cuboid its inner dimensions are :

5 m length, 3.5m width and 3 m hight. It is wanted to paint its lateral walls. The cost price of one square meter is LE 9. Calculate the required cost.

Solution :

The lateral area of the room's wall = Perimeter of the base x height

$= (\text{length} + \text{width}) \times 2 \times \text{height}$

$= (5 + 3.5) \times 2 \times 3 = 8.5 \times 2 \times 3 = 17 \times 3 = 51 \text{ m}^2$

The cost price = 51 x 9 = LE 459.

Exercise (3 – 4)

(1) Complet :

a) A cube of edge lenth 6 cm., Then its leteral area = ... cm^2.

b) The base area of a cube is 49 cm^2, Then its lateral area is .... cm^2.

c) The sum of the edge lenths of acube equals 84 cm, Then the length of the edge equals .... cm.

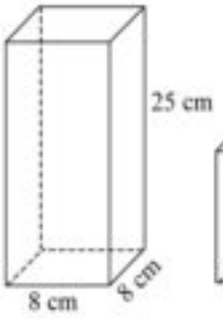
d) A hight a of cuboid which its total area is 120 cm^2 it dimensions of its base 4 cm and 6 cm caual ....

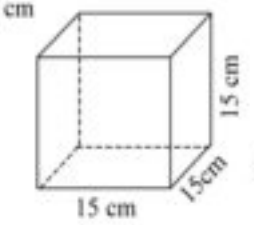
e) If the Lateral area of a cube 100 cm^2 then its total area = ... cm^2.

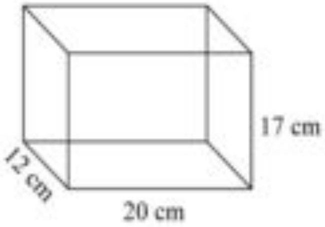
f) If the volume of a cube 1000 cm^2 then the its total area = ... cm^2.

h) If the perimeter of base of a cube is 24 cm then its total area = ... cm^2.

(2) Calculate the lateral area and total area for each of the following solids.







(3) Complete the following table (consider the unit length measured by cm).

Solid	Length	Width	Height	Lateral area	Total area
Cuboid	9.5	6	8	....	....
Cube	8	....	....	....	....
Cuboid	8.5	....	8	168	....
Cube	....	....	....	100	....

(4) If the lateral area of a cube is 36 cm^2. Find its total area.

(5) A cube of edge length 8 cm . Calculate the ratio between its lateral area and its total area.

(6) The total area a cube is 726 cm^2 . Calculate its lateral area.

(7) A cube of edge length 10 cm and a cuboid its length 8cm; its width 5 cm; its height 17 cm. Calculate the difference between their lateral area.

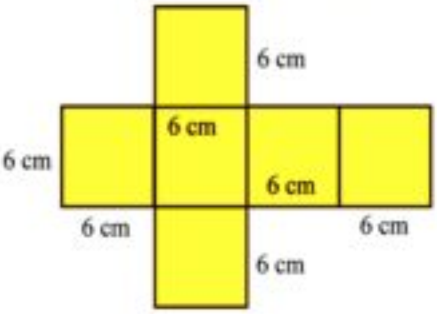
(8) A box without a lid its length 16 cm; its width 7 cm; its height 19 cm. Calculate its lateral area and total area.

(9) A box truck in the form of cuboid, its inner dimensions are 5 cm, 2.5m and 1.6 m.

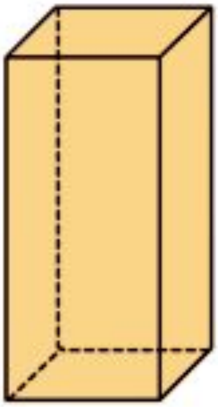
It is wanted to paint the inner box with paint, the cost price of one square meter is LE 12. Calculate the cost of paint.

(10) When folding the opposite shape,

- The solid formed is …………
- The lateral area of this solid is ………….
- The total area of this solid is ………..



(11) A cuboid shaped box with a square base its length is 9 cm and its height is 20 cm. Calculate the lateral area and total area.



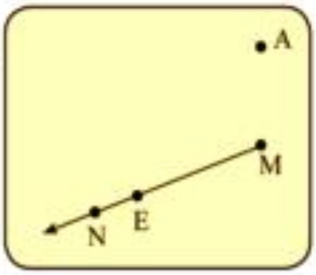
(12) A room its length is 5m, its width is 4m, and its height is 3.2m. It is wanted to paint its lateral walls and ceiling. The cost price of one square meter is LE 8. Calculate the required cost. Knowing that the room has 2 windows and a door their areas are $8m^2$.

(13) Youssef used a piece of cardboard in the form of rectangle, its length is 1.2m and its width is 80 cm to form a cubed box its edge length is 30 cm. Calculate the remained paper area after forming the box.

(14) The inner dimensions of a swimming pool are 30m, 10m and 1.5 m . It is wanted to cover it with a tile of squared shape its side length is 20 cm. If the cost price of one square meter is LE 32. Calculate the cost of covering its wall and ground.

(15) A box truck for carrying goods in the form cuboid, its inner dimensions are 4m, 2.5m and 1.8m. It is wanted to cover its side and ceiling with a sheet iron the cost price of square meter is LE 15. Calculate the cost of required sheet iron.

General Exercises on unit (3)



(1) From the opposite figure, Find the image of the point A by the translation $\overline{ME}$ in the direction of $\overrightarrow{MN}$.

(2) On the opposite coordinate plane:

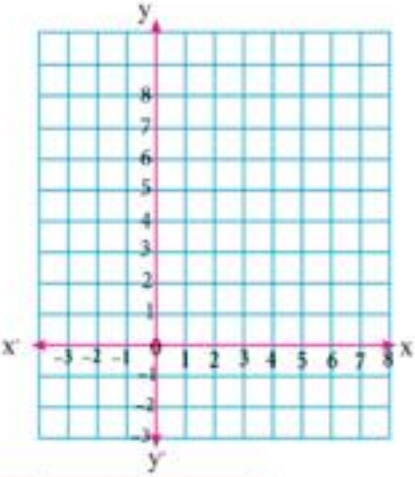
a- Determine the following points:
A (2, -2) , B (1, 1) and C (1, 6)

b- Find A′ which is the image of the point A by translation
(2, -1).

c- Find $\overline{B'C'}$ which is the image of $\overline{BC}$ by translation (3, 0).

d- Find BC and BB′.

e- Calculate the perimeter and the area for the shape BB′C′C.

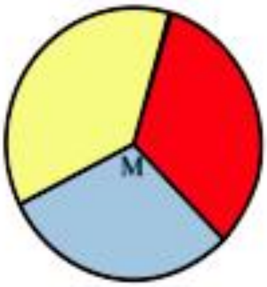


(3) Complete the following table :

The point	The translation	The image
(3 , 2)	(x + 3 , y + 1)	(...... , ...)
(...... , ...)	(x + 2 , y - 1)	(-3 , 3)
(0 , -3)	(x + ... , y + ...)	(0 , 0)
(-4 , -1)	(x + 3 , y + 1)	(...... , ...)

(4) A circle its circumference is 66 cm. Calculate its surface area. ($\pi = \frac{22}{7}$)

(5) In the opposite figure, a circle M of radius 7.7 cm is divided into three equal circular sectors, Find the surface area of one sector ($\pi = \frac{22}{7}$)

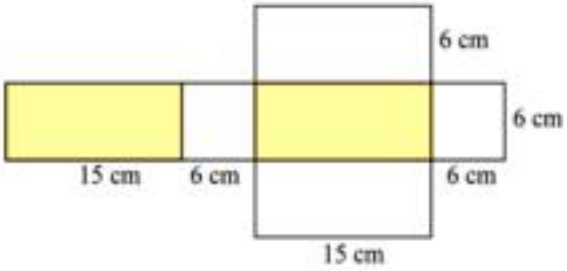


(6) A circular birthday tart of diameter 25 cm is divided into eight equal circular sectors, then find the surface area of one sector "Approximating the result to the nearest integer" ($\pi = \frac{22}{7}$ or 3.14)

(7) The perimeter of the base cube is 28 cm. Calculate its lateral area and total area.

(8) A container water tank in the form a cube its inner edge length is 1.5m, It is wanted to paint it to prevent the rust. The cost price of one square meter is LE 15. Calculate the cost of painting.

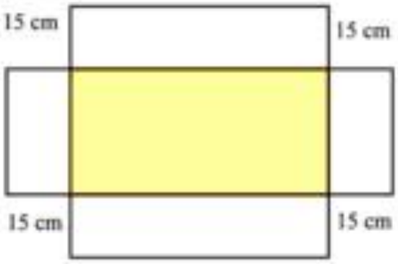
(9) When folding the opposite figure,
The solid formed is ....
The lateral area of the solid = ...
The total area of the solid = ...



(10) A room of squared ground its length is 4m and its height is 3m. It has a door its width is 90 cm, and 2m high. It has two rectangular windows equal in area of length 160 cm and width 100 cm. Calculate the cost of painting of the walls, given that the cost price of one square meter is LE 9.

Activity of the unit

Bring a sheet of cardboard (Bristol), then cut a square from each corner its side length is 15 cm to form the opposite figure fold the shape and use the glue to form the cuboid without a lid. Use your instruments to calculate its lateral area and its total area.



Technological Activity :

Subject of activity : Finding the lateral area and the total area of the cuboid by using "EXCEL program".

What do you learn from this activity : using "EXCEL program" in :

Entering set of data (length, width and height) for a cuboid through "EXCEL program".

Finding the lateral area and the total area of the cuboid by using the properties of "EXCEL program".

Example : Complete the following table to calculate the area :

The dimensions of cuboid			Lateral area	Total area
The length	The width	The height		
8	6	10		
10	10	3.5		
15	12.5	7		

The practical steps :

(1) Press "start", then choose program then choose "Microsoft Excel".

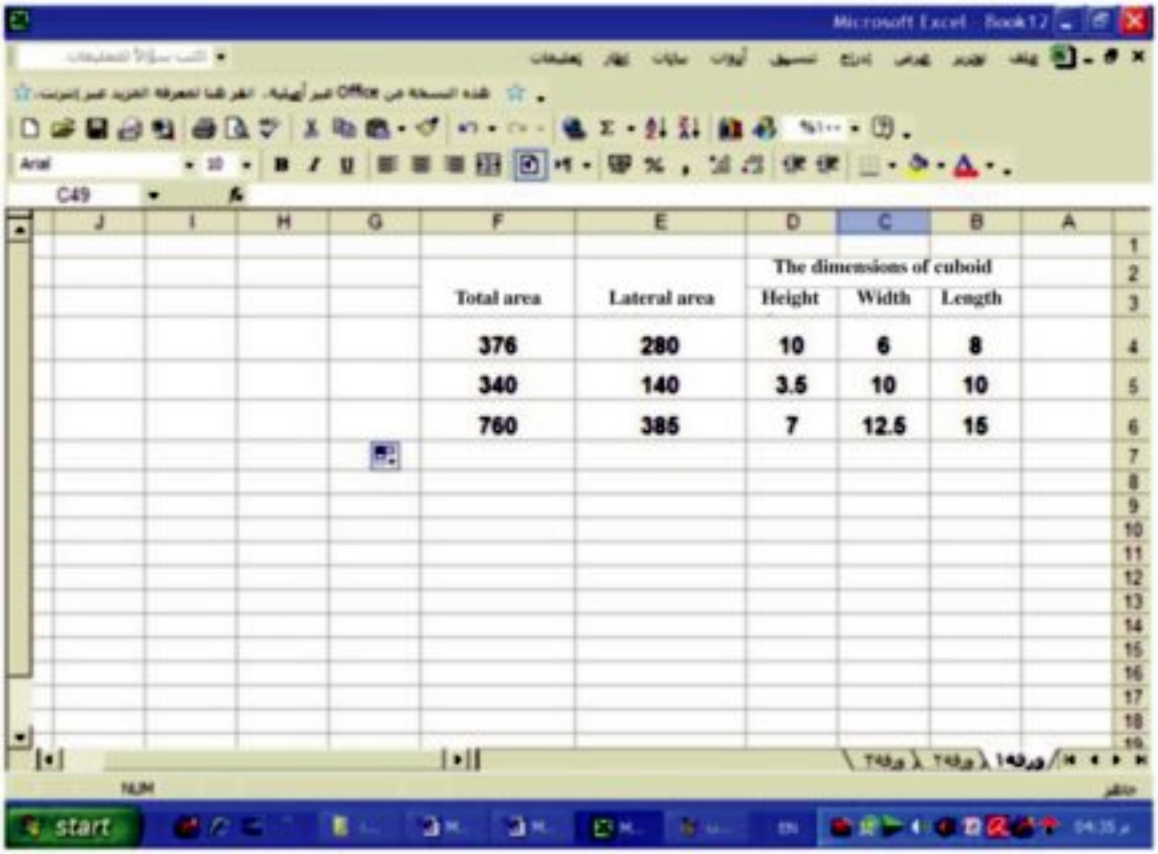
(2) Write the dimensions of each cuboid in the defined cells in the spread excel sheet.

(3) Write the dimensions of each cuboid in the defined cells (3) to calculate the lateral area and the total area of cuboid. Select E3 cell and write = ((B4 + C4) x 2) x D4 then press inter key.

(4) Select F3 cell and write = (B4 x C4) x 2 + E4 then press inter key.

By selecting the two cells E3 and F3 then drag down to the right corner at the end of the rows the results appear as the opposite figure.

Notice that: the sign of x on the keyboard is*



		The dimensions of cuboid		
Total area	Lateral area	Height	Width	Length
376	280	10	6	8
340	140	3.5	10	10
760	385	7	12.5	15

Unit test

(1) In the opposite figure,

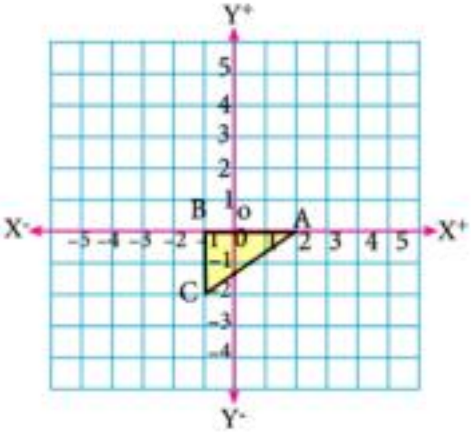
a- Dtermine the coordinates of the following points :

A (...... , ...) , B (...... , ...) and C (...... , ...)

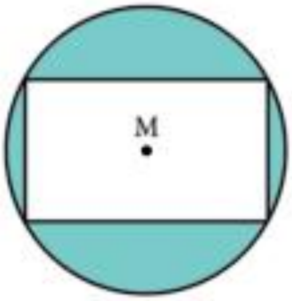
b- Find the image of the Δ ABC by translation $(x + 2 , y + 3)$.

c- The length of $\overline{BC}$ = ...

The length of $\overline{AB}$ = ...



(2) In the opposite figure, M is a circle of radius length 5 cm, A rectangle is drawn inside it, its length is 8 cm and width is 6 cm. Calculate the area of the shaded part. ($\pi = 3.14$)



(3) The sum of the edge lengths of a cube is 72 cm. Calculate its lateral area and total area.

(4) A room in the form of cuboid its inner dimensions are :

7m length, 5m width and 3.5m high. It is wanted to paint its lateral walls and the ceiling. The cost price of one square meter of paint is LE 11. Calculate the required cost.

Unit four

Statistics and probability

Lesson (1) : Representing the statistical data by using the circular sectors.

Lesson (2) : Random Experiment

Lesson (3) : Probability

- *General exercises on the unit*
- *Technological activity*
- *Activity of the unit*
- *Unit test*

1

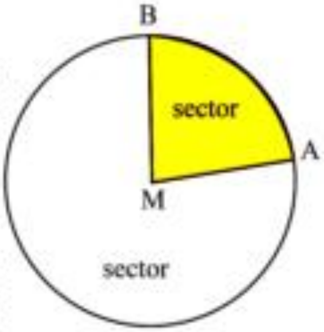
Representing the statistical data by using the circular sectors.

What do you learn from this lesson?
Through your active participation you can come to.
- dividing the surface of the circle into circular sectors.
- Calculating the measure of the angle of the sector.
- Representing the data by the circular sectors.

The mathematical concepts :
- Circular sector.
- The angle of the circular sector.

First: Dividing the surface of the circle into circular sectors.

Note and discuss :



The circular sector

Learn that the shaded part of the opposite surface circle represents the circular sector AMB.

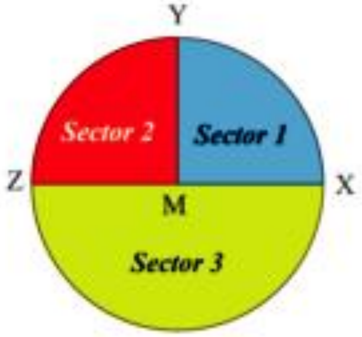
The shaded sector AMB is called a minor sector because its surface area is less than half the area of its surface circle.

The unshaded sector AMB is called a major sector because its surface area is greater than half the area of its surface circle.

Note that :

Angle of circular sector for every circular sector, there is an angle called "angle of the sector" and this angle is a central angle, where its vertex is at the centre of the circle.

Example (1) : Study the opposite figure, then complete :



-The surface area of the sector (1) $= \frac{1}{4}$ the surface area of the circle.

-The angle of the sector (1) is $\angle XMY$, its measure = 90°

- The surface area of the sector (2) $= \frac{1}{4}$ the surface area of the circle.

- The angle of the sector (2) is $\angle YMZ$, its measure = 90°

- The surface area of the sector (3) = $\frac{1}{2}$

- The surface area of the circle.

- The angle of the sector (3) is $\angle XMZ$, its measure = 180°

This means that the sum of the measures of the angles of the sectors about the centre of the circle. = 360^o

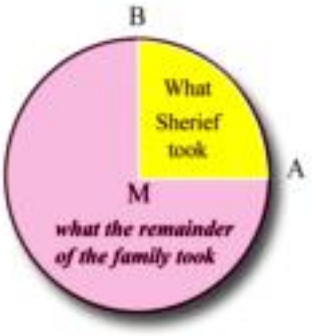
Remember that :
sures of the e
measures of the
accumulative angles
at a point" = 360^o

Example (2) :

Sherief took 25% from his birthday tart and he distributed the remainder of the tart among his family members, represent that using the circular sectors.

Solution :

The percentage of what Sherief took from the tart is 25% which represents $\frac{1}{4}$ of the tart, it can be represented by a sector, its area = $\frac{1}{4}$ of the surface area of the circle as shown in the opposite figure.



Note :

- All tart represents 100% of the surface area of the circle.
- Sherief' share is represented by the minor sector AMB.
- The share of the members of the family is represented by the major sector AMB of area $\frac{3}{4}$ of the surface area of the circle which is equivalent to 75% of the trat.

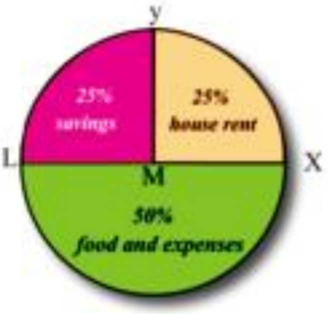
Example (3) :

Nahid is a clerk in on institution, she contributes with her husband by her Salary as follows : 25% for house rent, 50% for food and expenses and 25% for savings.

Note : Represent those data by using the circular sectors.

Solution :

In the opposite figure



- All the salary represents 100% of the surface area of the circle.
- The sector XMY which represents the house rent its surface area is $\frac{1}{4}$ the surface area of the circle represents 25% of the salary.
- The sector YML which represents the savings, its surface area is $\frac{1}{4}$ of the surface area of the circle represents 25% of the salary.
- The sector XML which represents the food and expenses, its surface area is $\frac{1}{2}$ of the surface area of the circle, represents 50% of the salary.

Note that :

In Examples (2) , (3) :

It is possible to represent the percentages 25%, 50% by using circular sectors easily because they represent $\frac{1}{4}$, $\frac{1}{2}$ of the surface area of the circle, also the measures of their central angles can be determined easily and they are (90° , 180°) respectively from 360°.

- Now the question is :

If the percentages are different from 50% ,25% as in drill (2) and it is required to represent these percentages by using the circular sectors.

This is what we shall learn together in the following:

Second : Representing the data by using the circular sectors.

Contribute and discuss.

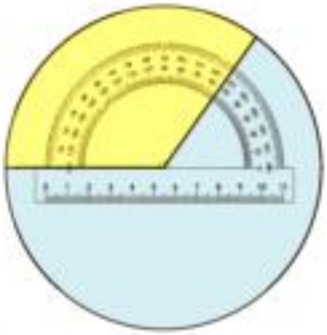
Example (1) :

The following table shows the percentages of the favourite subjects for the sixth primary in one of the schools through their questionnaire, represent those data by using the circular sectors.

Subject	Arabic	Mathematics	Science	Social studies
The percent of the number of the pupils	35%	25%	22%	18%

Solution :

Divide the surface of the circle into four sectors according to the given percentage in the table, where each sector represents one of the subjects after calculating the measure of each central angle for each sector, then draw these measures considering the sum of the measures of the central angles = 360° as the following :



1- Draw a circle M with suitable radius.

2- Calculate the measure of the central angle of each sector as the following :

- The measure of the central angle of the Arabic subject

$= \frac{35}{100} \times 360 = 126°$

- The measure of the central angle of the Maths subject

$= \frac{25}{100} \times 360° = 90°$

- The measure of the central angle of the science subject

$= \frac{22}{100} \times 360° = 79°$

- The measure of the central angle of the social studies subject

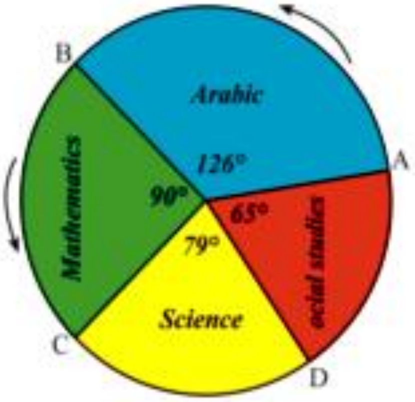
$= \frac{18}{100} \times 360° = 65°$

3- Draw the radius of the circle $\overline{MA}$ and consider it as a starting line for determining and drawing the first angle 126° by using the protractor to get the sector AMB (sector of the Arabic subject) as in the opposite figure .

4- Consider the radius $\overline{MB}$ as a starting line for determining the second angle 90° by using the protractor to get the sector BMC (sector of Maths subject).

5- Consider the radius $\overline{MC}$ as a starting line for determining the third angle 79° to get the sector CMD (sector of science subject).

6- Finally you get the remainder sector DMA which is the sector of social studies subject. Use the protractor to make sure that m ($\angle$ DMA) = 65° after you have finished these steps you will get the required data representation as the opposite figure.

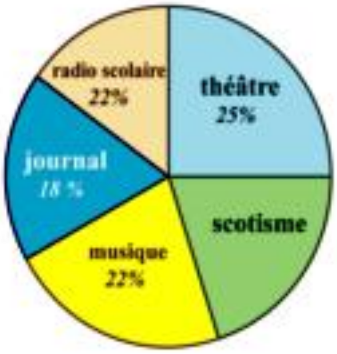


Note That :

All the measures of angles should be taken in one direction according to the arrows as illustruted in the figure.

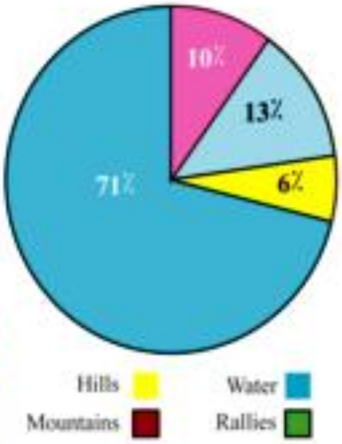
Exercises (4 - 1)

1- The opposite figure shows the favorite hobbies for the pupils of one of the classes in the sixth primary, study the figure, then answer



- What is the ratio of the theatre with respect to the remainder of the hobbies?
- What is the ratio of the broadcast with respect to the remainder of the hobbies?
- What is the ratio of the rangers with respect to the remainder of the hobbies?
- What is the measure of the central angle of the sector of the music?
- What is the hobbies that the least pupils prefer?
- What is the hobbies that the most pupils prefer?

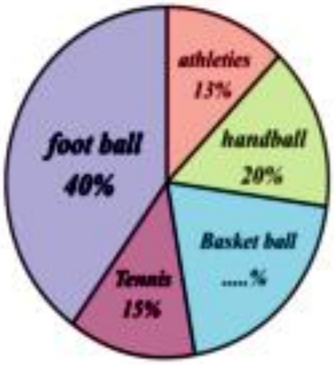
2- The opposite figure shows the distribution of the natural components of the earth's surface study the figure, then complete the following table.



The components of the earth's surface	Water natural supplies	Vallies	Hills	Mountains
Pourcentage	....	....	....	....

- What is the component which represents the smallest ratio of the earth's surface?
- What is the component which represents the greatest ratio of the earth's surface?
- What is the measure of the central angle of the sector of the vallies?

3 -The opposite figure shows the percentage of the favourite games for members of one of the sports clubs from their questionaire represented by the circular sectors.



Study the figure well, then complete the following:

- The ratio of the members who prefer the football is ....
- The ratio of the members who prefer the handball is ....
- The ratio of the members who prefer the tennis is ....
- The ratio of the members who prefer the basketball is ....
- The ratio of the members who prefer athletics is ....
- If the number of the club members is 2000, how many members prefer the handball?

4 - Five friends shared in a commercial business, with a capital LE 60 000, the first paid LE 12000, the second paid LE 6000, the third paid LE 15000, the fourth paid LE 9000, the fifth paid the remainder, illustrate that by using the circular sectors.

5- The following table shows the percentage of egg production in three farms, a marchent collected these eggs to distribute it on the grocery stores, represent these data by using the circular sectors.

The farm	Frist	Second	Third
The percentage of the production	25%	35%	40%

6 - The following table shows the percentages of production of a factory for three kinds of electric water heaters.

The kind	First	Second	Third
The percentage of the production	15%	30%	55%

Represent these data by circular sectors.

- If the total production in of the factory is 2000 heater, find the number of heaters of the second type

7 - One of the families spents its salary as the following 40% for food, 20% for house rent, 30% for expenses and save the remainder, represent these data by using the circular sectors, then answer the following.

If the family monthly in come is LE 900, so how much does the family save in the year?

- Another family spends its monthly salary by the same way and save LE 70 monthly, so what the monthly salary of that family?

8 - The following table shows the favourite TV programmes which the pupils of one of the classes in the primary six watch as the following.

Kind of programme	Entertaining	Cultural	News	Drama	Sports
Number of hours	9	5	4	7	11

Represent these data by using the circular sectors, then answer the following questions:

What is the programmes that the most of pupils prefer, also the least of pupils prefer?

2 Random experiment

What do you learn from this lesson?

Through your active participation you can come to:

- The concept of the random experiment.
- Calculating the outcomes for a number of random experiments.
- Solving miscellaneous exercises on calculating the outcomes for the random experiment.

The mathematical concepts

Random experiment.

- Sample space or (outcomes space).

Think and discuss :

One of the maths teachers showed his students in one of the classes of the primary six a coin (one pound) and they had this dialogue between him and his students.

The teacher : If a coin is tossed on the table or on the earth, what is the appearance face.

Adel : Either head or tail.

The teacher : well but why?

Adel : I am sure that the result is either head or tail, and not other wise.

The teacher : Who can determine the appearance face before tossing the coin.

Hanan : No one, but after tossing the coin we can be able to determine the appearance face.

The teacher : This means that, we can't predict (issuing a decision) that the result is either head or tail such experiment is called is a random experiment.

The random experiment : **It is an experiment in which we can determine all its possible outcomes before carrying it, but we can't predict in certainty which of these outcomes will occur when the experiment is carried out.**

Some examples for random experiments and their outcomes.

Random experiment	Possible outcomes
Tossing a coin once	Head (H), Tail (T)
Tossing a die once, observing the number of points on the upper face	1, 2, 3, 4, 5, 6
A ball is selected at radom from a box contains three symmetric balls (red, yellow, green)	Red, yellow, green
Carrying a game between your football team, other team from another school	Your team wins, your team is beaten, both of the two teams equalize

Sample space (outcomes)

Sample space — **The set of all possible outcomes for a random experiment.**

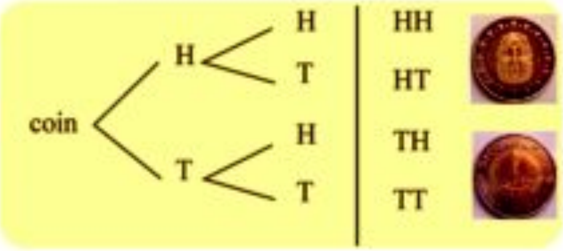
Notice : From the previous experiments.

- Sample space for tossing a coin once { = H, T }
- Sample space for tossing a die once. {6, 5, 4, 3, 2, 1} =

Some examples for random experiments, their corresponding sample spaces.

Example (1) :

The random experiment is tossing two distinct coins once, find the sample space.



Solution :

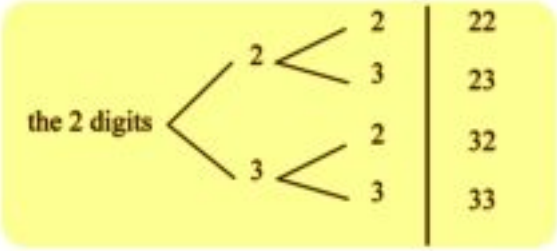
The sample space from the opposite figure is {HH, HT, TH, TT} where HH means that the result of tossing the coins is :

The first coin is head, second is head, HT means that the result of tossing the coins is the first is head, second is tail and so on ....

> *Notice that :*
>
> **Tossing two coins once is equivalent to tossing a coin two consecutive times, and so on ......**
>
> **Tossing two dice once is equivalent to tossing a die two consecutive times, and so on ....**

Example (2) :

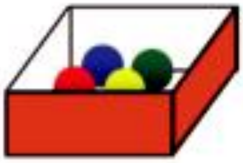
The random experiment is getting a 2-digit number using the digits 2,3



Solution :

The sample space from the opposite figure is {22 , 32 , 23 , 33}

Example (3) :



The random experiment is : selecting a ball from a box contains 4 symmetric balls (red, yellow, greem blue) write the sample space to know the colour of the selected ball.

Solution :

Sample space is : {red, yellow, green, blue).

If the random experiment is selecting a card from :

Drill (1) :

A box contains 9 equal cards, have the same colour, numbered from 1 to 9. Write the sample space for this experiment.

Drill (2) :

The random experiment is tossing a coin two consecutive times under the condition appearance of tails only. Write the sample space for this experiment - what is the number of tails in this case ?

Exercises (4 – 2)

(1) Complete : The random experiment is ………………………………

- The sample space is ………………………………

(2) If the random experiment is tossing a coin two consecutive times under the condition : "appearance of a head only. Write the sample space for this experiment".

(3) If the random experiment is visiting one of your relatives to know the gender of the child product by his wife.

Write the sample space of this experiment.

(4) In the random experiment of tossing a coin two consecutive times to know the appearing face, Write the sample space.

(5) In the experiment of selecting a ball from a box contains 3 red balls, 4 yellow balls all of them are equal in volume, observing the colour of the selected ball, write the sample space of this experiment.

(6) In the experiment of tossing a die under the condition "the number of dots on the upper face is an odd", write the sample space.

(7) In the experiment of tossing a die under the condition "the sum of the dots on the upper two faces is 7", write the sample space.

3 *The probability*

What do you learn from this lesson?
Through your active participation you can come to :
- Writing the sample space of a random experiment.
- Determining the elements of the random experiment.
- Determining the concept of the event.
- Calculating the probability of an event inside a random experiment.

The mathematical concepts:
- Sample space.
- Event
- probability of the event.

Notice and discuss:

We have studied in the previous lesson the sample space of a random experiment we knew that the sample space is the set of all possible outcomes for a random experiment.

- We denote to the sample space by (S), its elements number by n (S).

Example (1) :

In the experiment of tossing a regular coin and observing the appearing face, set of sample space is S = {H, T}, n (S) = 2

Example (2) :

In the experiment of tossing a regular die, observing the number appearing on the upper face, set of sample space is S = {1, 2, 3, 4, 5, 6}, n (S) = 6.

Example (3) :

A cord is drawn from 5 symmetrical cards numbered from 1 to 5 without looking at them, so the sample space = {1, 2, 3, 4, 5},
n (S) = 5.

Event: **Any outcomes you can get inside a random experiment are called events.**

Example (4) : Tossing a regular die once, observing the number appearing on the upper face, consider the following events :

The event (A) : appearance of an even number on the upper face.

The event (B) : appearance of an odd number on the upper face.

Solution :

Sample face S = {1, 2, 3, 4, 5, 6} , n (S) = 6
Event A = {2, 4, 6}, n (A) = 3,
Event B = {1, 3, 5} , n (B) = 3

Notice : **$A \subset S$, $B \subset S$, so we deduce that:**

The event : **It is a subset of the set of sample space, the number of its elements represents number of times of its occurrence.**

The ratio between the number of elements of an event and number of elements of the sample space is called the probability of occurrence of the event, more abbreviation : (probability of the event and is denoted by "p").

From the previous example :

$$P(A) = \frac{\text{number of elements of the event (A)}}{\text{number of elements of the sample space}}$$

$$= \frac{n(A)}{n(S)} = \frac{3}{6} = \frac{1}{2} = 0.5 = 50\%$$

$$P(B) = \frac{\text{number of elements of the event (B)}}{\text{number of elements of the sample space}}$$

$$= \frac{n(B)}{n(S)} = \frac{3}{6} = \frac{1}{2} = 0.5 = 50\%$$

Notice:

It is possible to add other events through the previous example like :

1- The event (C) is :

Appearance of a number less than 3 on the upper face on the die.

Then $C = \{1 , 2\}$, $n(C) = 2$

, then $P(C) = \dfrac{n(C)}{n(S)} = \dfrac{2}{6} = \dfrac{1}{3} \simeq 0.33 \simeq 33\%$

2- The event (D) is :

appearance a number greater than 6 on the upper face of the die. This is the impossible event (does not occur) Why?

Then $D = \varphi$, $n(D) = 0$

Then $P(D) = \dfrac{n(D)}{n(S)} = \dfrac{0}{6} = 0$

3- The event (E) is : appearance of a number less than 7 on the upper face of the die.

This is the sure event (Its elements are all the possible outcomes of the experiment).

Then $E = \{1, 2, 3, 4, 5, 6\}$, $n(E) = n(S)$

Then $P(E) = \dfrac{n(E)}{n(S)} = \dfrac{6}{6} = 1$

From the previous, we deduce that :

The event (A) inside the sample space has 3 types :

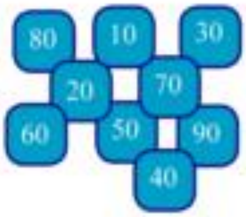
1 - The impossible event (can not occur), expressed by $A = \varphi$, probability of its occurrence $P(\varphi) = 0$.

2 - Sure (Certain) event (all possible outcomes), then $A = S$, probability of its occurrence is $P(S) = 1$.

3 - Possible event (some of the out comes of the experiment), then $A \subset S$, the probability of its occurrence = proper fraction, this means that the value of the probability of the event (A) where $A \subset S$ is not less than zero, not greater than 1, so the following inequality is satisfiered $0 \leq p(A) \leq 1$.

Example (5) :

A box contains 9 symmetric cards each carries a number from the numbers (10 to 90) they are mixed well, then one card is selected at random find the probability of the following events.



1 - The event A, where A is a number that is divisible by 5.

2 - The event B, where B is a number that is divisible by 3.

3 - The event C where C is an odd number.

Solution :

Samplespace is $S = \{10, 20, 30, 40, 50, 60, 70, 80, 90\}$, $n(S) = 9$.

- The event $A = \{10, 20, 30, 40, 50, 60, 70, 80, 90\}$ et $n(A) = 9$

$$\text{, then } P(A) = \frac{\text{number of elements of the event (A)}}{\text{number of elements of the event (S)}}$$

$$= \frac{n(A)}{n(S)} = \frac{9}{9} = 1 \qquad \text{(certain event)}$$

- The event $B = \{30, 60, 90\} \subset S$, $n(B) = 3$

$$\text{The } P(B) = \frac{\text{number of elements of B}}{\text{number of elements of S}}$$

$$\frac{3}{9} = \frac{1}{3} \simeq 0.33 = 33\%$$

- The event $C = \varphi$ (Impossible event) then $n(C) = 0$

$$\text{Then } P(C) = \frac{n(C)}{n(S)} = \frac{0}{9} = 0$$

Example (6) :

In the ideal student competition of one of the schools 63 boys, girls applied, for the competition if the probability that one of the girls is an ideal student is $\frac{4}{9}$, find the number of girls who participated in the competition.

Solution :

The total number of the applicant students = 63 let the event A is that one of the girls is an ideal student.

Then $P(A) = \frac{4}{9}$ (given in the problem)

But $P(A) = \frac{\text{number of girls}}{\text{total number of the student}} = \frac{4}{9}$

$\frac{\text{Then number of girls}}{63} = \frac{4}{9}$ (proportion property)

Number of girls $= 63 \times \frac{4}{9} = 28$ girl

Notice:

1 - The probability can be written as a fractional form or decimal form or in the form of percentage.

2 - If the results of the experiments are known previously, so they are not random experiments.

For example :

- The experiment of selecting a ball from a box contains 3 symmetrical red balls.

- The experiment of selecting a card from a box contains 5 symmetrical cards each of them carries the number 10.

- The experiment of selecting T-shirt from a box contains 20. T-shirts each of the same size, same colour.

Exercises (4 – 3)

(1) Choose the correct answer between brackets :

(a) If ϕ is empty set then p (ϕ) = { 0 , 2 , 1 , 0.5 }

(b) If s is a sample space of a vandom experiment, then p (s) = ...

(zero , 2 , 1 , 0.8)

(c) If a dice is tossed once, then probability of getting an even number = ...

(zero , 2 , 1 , 0.5)

(d) When tossing adie once, then Probability of getting a number divisable by 3 equals

= ....

(zero , $\frac{1}{3}$, $\frac{1}{2}$, 1)

(e) When tossing a coire once, then Probability of getting a head is ....

(zero , 2 , 1 , 0.5)

(2) In the experiment of selecting a card at random from 7 equal cards numbered from 1 to 7, write the sample space, then find the probability of :

- The event A, where A is appearing a number less than 4.
- The event B, where is appearing of an odd number.
- The event C, where C is appearing a number more than 5.

(3) If the experiment is "A student is chosen at random from a class of 40 students, 32 students have succeeded in Maths test, 35 students have succeeded in Arabic test find the probability of :

- The event A, where A is the event that he has succeeded in Arabic.
- The event B, where B is the event that he has succeeded in Maths.
- The event C, where C the event that he has failed in Maths.

(4) In the experiment of tossing a regular die once and observing the number of dots on the upper face, find the probability of :

- The event A, where A is the event of appearance of a number less than 5.
- The event B, where B is the event of appearance of a number satisfies the inequality $B \geq 3$.

(5) In one of the "weight loss" centres, 10 ladies suffering from over weight were waiting to enter for meeting the specialized doctor if the weights of 4 them are between 100, 110kg, the weights of the others are between 110, 120 kg, find the following probabilities :

- Entrance of a lady of weight less than 110kg.
- Entrance of a lady of weight more than 110kg.
- Entrance of a lady of weight 90kg.

(6) A box contains 8 white balls, 12 red all of them are symmetric, a ball is selected without looking inside the box, find the following probabilities :

- The selected ball is white.
- The selected ball is red.
- The selected ball is blue.

(7) In the experiment of forming a 2-digit number from the digits {3, 5}. Write the sample space, then find the probability of the following events.

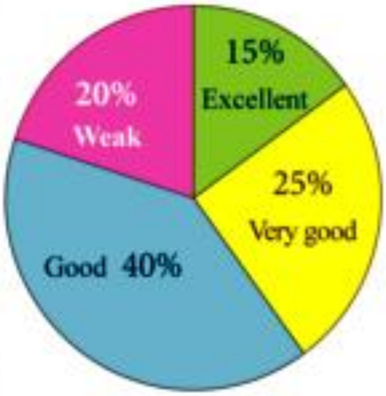
- The event A where A is unit digit equals the tens digit.
- The event B where B is the tens digit is an odd digit.
- The event C where C is the unit digit is an even digit.

(8) In the experiment of tossing a die once, observing the number appearing on the upper face write the sample space, then find probability of the following event A, where A is the event of getting a number ≤ 3.

General Exercises on unit (4)

(1) The following figure represents the grades of 40 students in mathematics exam. Fill the following data the in the opposite table, then calculate the measure of the central angles for each grade.

The grade	The percentage	Number of the students	The measure of the central angles
Excellent			
Very good			
Good			
Weak			
The sum			



(2) The following table shows the percentages of nutrients which the pizza contains as the following:

Components	Protein	Sugar	Corn flour	Fats	Vitamins
The percentage	11%	14%	37%	13%	25%

Represent the previous data by using the circular sectors.

(3) The following table shows number of hours that Nahed spent for revising the different subjects weekly :

Subject	Arabic	English	Maths	Science	Social studies
Number of hours	9	6	7	5	9

- Represent the previous data by using the circular sectors.

(4) If the random experiment represents visiting one of the families which have two children to know the gender of the children. Write the sample space for this experiment.

(5) In the experiment of forming a 2- digit number from the set of digits { 5 , 6 }. What is the probability :

- The event A : the unit digit of A is an odd number.
- The event B : the sum of the two digits is 11.
- The event C : the two digits are equal.

(6) In the experiment of choosing two students of your class for participating one of them to play tennis in the school team. The first student throws the ball 10 times and scored 4 of them, the second student throws the ball 12 times and scored 6 of them. Determine which of the two students that the coach chooses and why ?

(7) A box contains 10 cards numbered by the even numbers from (2 to 20) one of the cards is selected at random. Calculate the probability of :

- The event A : appearance of the multiples of number 4.
- The event B : appearance of an even number.
- The event C : appearance of a number that is divisible by 3.

(8) A box contains 25 colored ball, 13 red, 12 yellow. If one ball is selected from the box at random. Calculate the probability of :

- The event A : the selected ball is red.
- The event B : The selected ball is yellow.

Technological Activity :

Subject of activity ; Using the "EXCEL program" for representing data by using circular sectors.

What do you learn from this activity : using "EXCEL program" in :

Entering set of data "ECEL program".

Representing data by using circular sectors using the properties of the "EXXEL program".

Activity :

The following tables shows the percentages for a number of students in one of the classes according to their favourite activities use the "EXCEL program" to represent these data by using the circular sectors.

The activities	Computer	English	Reading	Music
The percentage	35%	25%	27%	13%

The practical steps :

(1) Press "start", then choose program then choose "Microsoft Excel".

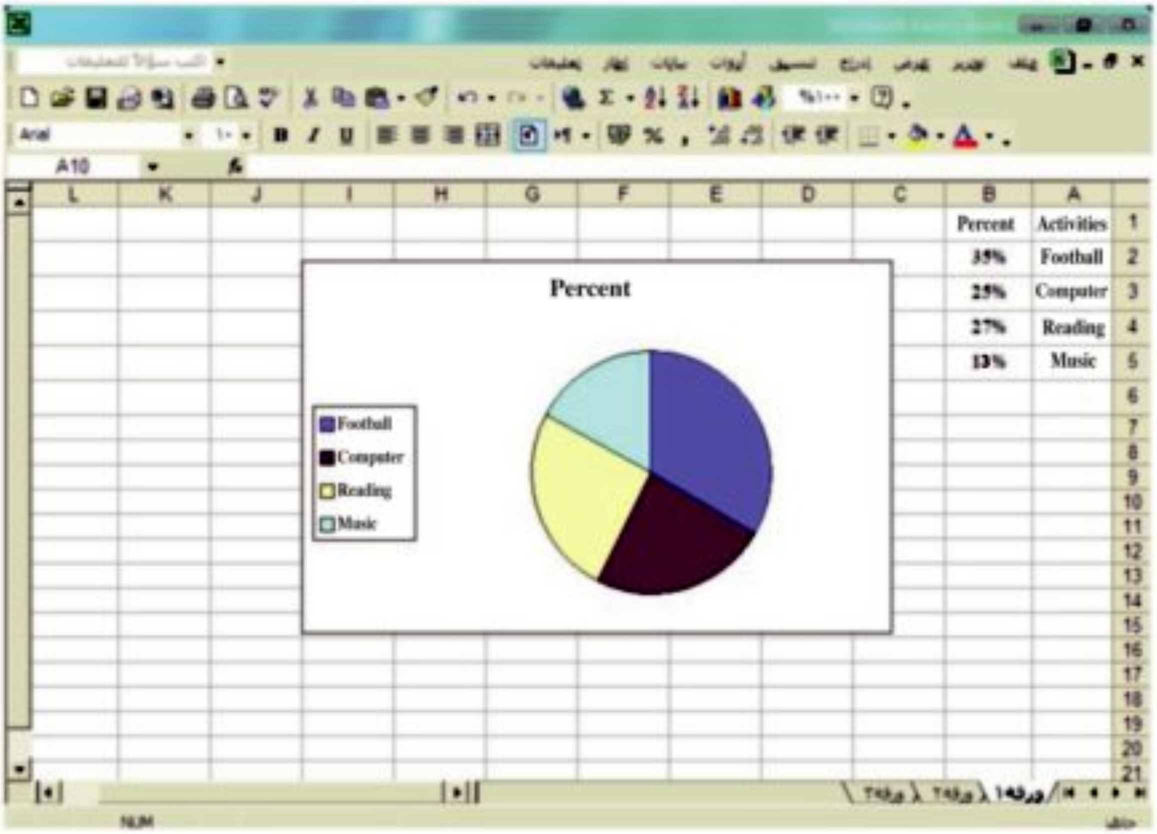
(2) Write the data in the first row for the previous table (The activity) in the cells of column A respectively.

(3) Write the data in the second row for the previous table (The percentage) in the cells of column B respectively.

(4) Select the data for the percentage of the students in the two coloumns A and B by using mouse.

(5) Choose "Chart" from the insert menu by clicking mouse on it.

(6) Choose pie then press on it, then press finish. The following chart will appear :



Activities	Percent
Football	35%
Computer	25%
Reading	27%
Music	13%

1- Help your colleague for a survey to know the favourite fruits for each person in the class from the following types (Oranges – Banana – Guava – Dates – Watermelon). Display the given data in a simple frequency table.

Represent these data by using the circular sectors.

2- Toss a coin 30 times; record what you have got in the following table .

- Calculate the probability of the event A where A is the event for appearance a head.
- Calculate the probability of the event B where B is the event for appearance a tail.
- What is your expectation about the chance of appearing the head or tail if the number of tossing is increasing to :

100 times – 500 times – 1000 times.

The event	The tally	The frequency
Head		
Tail		
The sum	30	

3- By using the cardboard, cut 10 squared or rectangular equal cards and have the same color, then write a number in each one of them from the digits (1 to 10), then put them in a non appearance bag and mix them carefully, choose one of them at random.

Calculate the probability of the following events :

- The event A : appearance of a number more than 7.
- The event C : appearance of an odd number.
- The event B : appearance of a number that satisfies the inequality $B \leq 10$.
- The event D : appearance of a number that satisfies equation $D - 4 = 2$.

Unit test

(1) The following table shows the percentage of the favourite sport for your class students.

The favourite sport	Football	Basketball	Volleyball	Swimming	Ping-Pong
The percentage	45%	9%	24%	10%	12%

Represent the previous data by using the circular sectors.

(2) In a meeting for discussing the problems of the workers in a factory, 100 workers were attending from men and women. If the probability of a man for standing to show the problems of the workers is $\frac{3}{5}$. Calculate the number of the men and women in this meeting.

(3) In one of the classes of the sixth primary, the teacher of maths classified the levels of the students in his subject into (weak – intermediate – advanced) their number is 40 students and recorded his data in the opposite table :

The level	Number of the students
Weak	5
Intermediate	25
Advanced	10
The sum	40

One of the students in this class is chosen at random.

Calculate the probability of :

- The event A : where A is a weak student.
- The event B : where B is an advanced student.
- The event C : where C is not an intermediate student.

(4) A fair die is rolled once, and the number of dots on the upper face is observed. Calculate the probability of :

- The event A : where A is the appearance of a number less than 4.
- The event B : where B is the appearance of a number that satisfies the inequality $1 < B < 6$.

(5) A class of 40 students has got a maths exam its maximum mark is 50, if 30 students got less than 40 marks, and 10 students got (40 up to 50) marks. Calculate the probability of :

- The event A : where A is a student has got less than 40 marks.
- The event B : where B is a student has got a mark satisfies the inequality $B \geq 40$.

Answers of the general tests of the units, model tests (second term)

Test of unit (1) Integers

(1) (a) N (b) $Z^+ \cup \{0\} \cup Z^-$

(c) one (d) $Z = N \cup Z^-$.

(e) φ (f) - 54

(g) $\subset$

(h) $\subset$

(2) 12, 7, 0, -9, -15

(3) a) 10 b) -2

(4) a) -12 b) 44

(5) 22°C

(6) (a) 9

(b) 1

(c) -256

(7) 8 months, the pattern is {75, 72, 69, ..., 51), description of the pattern each number decreasers 3 than the previous one.

(8) Numerical pattern = {1, 3, 6, 10, 15, ...}, its formula each number is more than the previous by its order.

The test of unit (2)
(equations and inequalities)

(1) left to the student.

(2) (a) -2 (b) -1

(c) 6 (d) -1

(3) (a) φ

(b) {2, 1, 0, -1, -2, ...}

(4) (a) {-2} (b) φ

(b) {3} (d) {-3}

Test of unit (3)
Gemetry and measurement

(1) (a) (2, 0) (b) (-1, 0) , C (-1, -2)

(c) BC = 2 cm , AB = 3 cm

(2) 46.5 cm^2

(3) 144 cm^2, 216 cm^2

(4) L.E 1309

Test of unit (4)
Statistics, probability

(1) left to the student

(2) 60, 40

(3) $P(A) = \frac{1}{8}$, $P(B) = \frac{1}{4}$, $P(C) = \frac{3}{8}$

(4) $P(A) = \frac{1}{2}$, $P(B) = \frac{2}{3}$

(5) $P(A) = \frac{3}{4}$, $P(B) = \frac{1}{4}$

Model (1)

Answer the following questions :

The First Question : Choose the Correct answer from those given :

1) $(-1)^8 + (-1)^9 =$ .... (zero , -1 , 1 , 2)

2) The image of the point (-3, 4) by the translation (x , y - 4) is ....

((-3,0) , (-7,4) , (-3,8) , (-1,4))

3) $\{0\}$ .... N ($\in$, $\notin$, $\subset$, $\not\subset$)

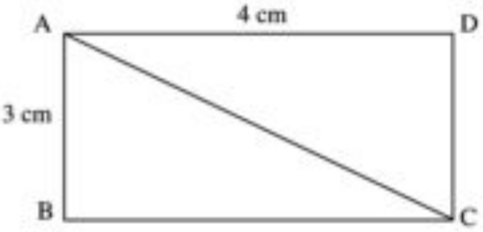
4) When tossing adie once, then probability of getting a number on the upper face more than 6 = ... ($\varnothing$, zero , $\frac{1}{6}$, $\frac{1}{3}$)

The Second Question : Complete the following :

1) $\left|\frac{5-11}{3}\right|$ .... Z by using ($\in$, $\notin$, $\subset$, $\not\subset$)

2) If $x + 6 = 2$, $X \in Z$, then X = ...

3) In the opposite figure ABCD is a rectangle, Then the area of Δ ABC = ... cm^2.



4) A box Contains 5 white balls, 3 blue balls and 8 red balls all of them are Symmetric. one ball is drawn from the box at random. Then the probability that the drawn ball is red = ...

The Third Question :

A) Find the rosult of $(4 \times 3^2 \div 3^2 - 7 \times 3)$

B) Find the solution set of the inequality $X - 2 \geq 3$, $X \in Z$

The Fourth Question :

A) A cubold shaped box with a square base its length is 10 cm and its height is 7 cm. Calculate the lateral area.

B) The circumference of a circle is 88 cm. Calculate its area.

The Fifth Question :

A) Find the solution set of the equation :

$3X + 9 = 3 \quad , \quad X \in Z$

B) The following Table shows the percentage of the production of a factory of house electrical sets.

The kid of set	washig machine	heater	oven	mixer
The percentage	30%	15%	40%	15%

Represent these data by circular sectors.

Model (2)

Answer the following questions :

The First Question : Choose the Correct answer from those given :

1) If $2x = 6$ Then $X \in$ .... $(N , \varnothing , Z_+ , Z_-)$

2) The Circumference of the clrcle = ... X π $(r , 2r , r^2 , r+2)$

3) When Tossing a die once, then probability of getting a number 5 = ...

$(\text{zero} , \frac{1}{6} , \frac{5}{6} , 1)$

4) The number which satisfies the inequality $X - 2 > 3$ is ....

$(-1 , -2 , -3 , -4)$

The Second Question : Complete the fellowing :

1) $\frac{2^3 \times 2^5}{2^2} =$ ....

2) The counting numbers (C) .... N

3) A cube of total area 150 cm^2, then the length of its edge is .... cm.

4) In a 6th primary class, the marks of the students are given in the following table.

If one of students is randownly chosen, then the probability that this pupil got good estimate is ....

Excellent	very good	good	weak
8	18	16	6

The Third Question :

A) Find the result of $(6 \times - 5 - (2 \times 3) \div 3)$

B) Find the solution set of the inequality $X - 2 \geq 3$ where $X \in Z$ then represent it on the number line.

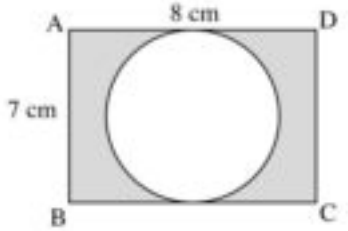
The Fourth Question :

A) Find the solution set of the equation

$2x + 9 = 5$ where $X \in Z$

B) In the opposite figure ABCD is a rectangle where its length = 8 cm and its width = 7 cm

Calculate the area of shded part.



The Fifth Question :

A) In a cartesian co-ordinate plane locate the points A(2,3) , B(4,3) , C(4,7), then find.

(1) The length of $\overline{BC}$ = ... units.

(2) The image of Δ ABC by the translation (0, -4)

B) The following table shows the number of students participating in the school activties.

The activity	Cultural	Sports	Social	Arts
The percentage	5%	45%	15%	35%

Represent these data by circular sectors.

(For the special needs)

Model (3)

Answer the following questions :

The First Question : Complet the following :

1) $|3| =$ ....

2) The probability of the impossible event = ...

3) If $X + 2 = 3$, $X \in N$, Then $X =$ ....

4) The perimeter of the base the Cuboid is 10 cm, its height is 4 cm. Then its lateral area = ... cm^2.

The Second Question : Choose the correct answer from those given :

1) $2^5 \times 2^2 =$ .... $(2^7 , 4^7 , 1)$

2) The surface area of a circle = $\pi \times$ .... $(r , r^2 , 2r)$

3) $Z_+ \cup \{0\} =$ .... (Z_- , N , Z)

4) When Tossing a die once then probability of getting an odd number = ...

$(\frac{1}{6} , \frac{1}{3} , \frac{1}{2})$

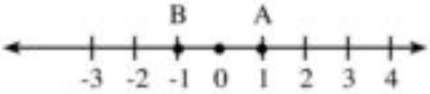
The Third Question : Put true (✓) or false (×) :

1) $|-5| + 5 = 10$ ( )

2) If $3x = 9$, Them $X = -3$ ( )

3) The probablity of the sure event = zero ( )

4) In the opposite figure the distance between points A and B = 2 units ( )



The Fourth Question : Join from column (A) to column (B) :

	(A)	(B)
1)	The sum of the measures of the angles of the sectors a bout the centre of the circle = ...	$\in$
2)	2 .... Z_+	360°
3)	The solution set of the inequality $x + 2 < 5$ then $X \in N$ is	(4 , 4)
4)	The image of the point (3,2) by the transtation (1,2) is	{ 0 , 1 , 2 }

The Fifth Question Complete the following :

A) The length of the edges of a cube is 4 cm Calculate its total area and lateral area

The Total area = 6 × ....

The lateral area = 4 × ....

B) Find the result :

$$\frac{2^3 \times (-2)^4}{2^5}$$

$$\frac{2^3 \times 2^4}{2^5} = \frac{2^{\cdots + \cdots}}{2^5} = 2^{\cdots} = \ldots\ldots\ldots\ldots$$

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